

Dynamic Legislative Bargaining with Endogenous Agenda Setting Authority

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Abstract

Models of repeated legislative bargaining typically assume an agenda setter is randomly selected each period, even if the previous period agenda setter successfully passed a proposal. In reality, successful legislative agenda setters (e.g., speakers, committee chairs) tend to hold onto power. We propose two alternative models in which successful agenda setters retain power. In the first model, a successful agenda setter automatically keeps power. Such an assumption is easy to work with and results in a policy equal to that in a traditional non-repeated game. In the second model, an agenda setter requires the support of a legislative majority to retain power. Such an assumption is realistic and results in the most-equitable policy outcome. Compared to both of these models, the standard random-selection model exaggerates the agenda setter's ability to extract rent from the legislative process, and underestimates the wellbeing of the legislative majority. (*JEL*: D72, D78. *Keywords*: repeated legislative bargaining, stationary equilibrium, agenda control, proposal power)

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1 Introduction

In their influential paper, Baron and Ferejohn (1989) adapt a Rubinstein (1982) bargaining framework for legislative decision making. A random legislator is appointed as the agenda setter to propose a division of a given budget. The other legislators then vote on the proposed budget. If a majority of legislators vote in favor of the proposal, it passes and the game ends. Otherwise, a new legislator is randomly selected to make a new proposal and the process repeats. Our article presents a framework in which legislators repeatedly engage in a Baron-Ferejohn legislative bargaining game. When a proposal passes in our framework, the game does not end; rather, the legislature begins work on the next budget allocation. Within each round of bargaining—from the period following the passage of one proposal through the period in which the next proposal passes—the game is identical to the closed-rule bargaining game in Baron and Ferejohn (1989).

This is not the first article to model legislative bargaining in a repeated environment. A growing literature applies such a framework to analyze how policy evolves over time (e.g. Baron 1996, Kalandrakis 2004, 2009, Baron and Herron 2003, Battaglini and Coate 2007, 2008, Duggan and Kalandrakis 2010). In focusing on how the status quo changes over time, the articles make simplifying assumptions regarding the transition of agenda setting power from one round of bargaining to the next. With few exceptions, the literature assumes that a new legislator is randomly selected to serve as agenda setter each period, independent of past success. Even a successful agenda setter with the support of most other legislators is unable to hold onto power. Frequent turnover of agenda setting power is built into the models. Such an assumption is inconsistent with the fact that agenda setting authority in the US Congress rarely changes hands. Since the first US Congress in 1789, for example, there have been only 59 changes in the Speaker of the House. No more than 24 of these changes can be attributed to the speaker losing support amongst his party.¹ Committee chairmanships show similar consistency. In the US Senate, for example, there have been 26 different chairmen of the appropriations committee since its establishment in 1867.² The average Speaker of the House held the position for 4.3 years, and the average appropriations committee chairman held the position for 5.5 years.

We propose two alternative assumptions about if and how a successful agenda setter can hold onto power. The models involve endogenous proposal power in that an agenda setter’s ability to stay in power may depend on her current-period proposal. The simplest of the two frameworks assumes that an agenda setter who successfully passes a proposal in period t automatically serves as the agenda setter again in period $t + 1$. The model has a number of attractive features. First, it

¹Of the changes, 28 corresponded to a change in the House majority party, four were due to the death of the sitting speaker, one was due to Schuyler Colfax stepping down to become vice president, and two were due to Henry Clay temporarily stepping down to dabble in international relations. Our estimate of 24 speakers who may have lost the support of their party as including the 23 changes in speaker that took place between sessions of congress in which the same party maintained power, and when James Wright stepped down mid-congress amid an ethics scandal.

²Similarly, there have been eight different chairmen of the budget committee since it was established in 1975, and 19 different chairmen of the banking committee since its establishment in 1913 (either the Committee on Banking and Currency that existed from 1913 to 1970 or the Committee on Banking, Housing, and Urban Affairs that has been in place since 1970).

is no more complicated to work with than the standard random-agenda setter assumption. Second, consistent with the realities of the legislative process, the model predicts low turnover of agenda setter power in equilibrium. Third, compared to the random-agenda setter assumption, the model gives less bargaining power to the agenda setter and more to the legislative majority. This means that the model results in less extreme predictions about the ability of the agenda setter to extract rent from the legislative process, and less pessimistic predictions about the wellbeing of the legislative majority. Interestingly, the model results in a per-period allocation that is identical to the one-time allocation in the non-repeated Baron and Ferejohn game. This suggests that the non-repeated legislative bargaining game, and the results that come out of such models, may not unravel when we allow for repeated interactions. In fact, depending on the specifics of the repeated process, the predictions of the repeated and non-repeated models may be identical.

The second framework we propose involves the legislature first voting on a proposal, then voting on whether to keep or replace the agenda setter. In order to keep power, the agenda setter must maintain the support of a majority of other legislators. Passing a proposal is not enough to stay in power. This model is the most realistic framework that we consider; it is also the most complicated. If legislators do not care enough about future periods, then the equilibrium resembles the equilibrium under the random-agenda setter assumption. That is, the agenda setter maximizes her current period rent extraction without concern for staying in power. The more interesting equilibrium exists when legislators care enough about future periods (a standard assumption through much of the literature). In this case, the agenda setter shares a large enough portion of the budget with other legislators in order to maintain their support, and hold onto power. Compared to the standard model, the agenda setter has even less bargaining power in this framework, and is able to extract even less rent from the legislative process. The legislative majority does not have to give up an entire period's allocation in order to remove the agenda setter from power if she does not keep them happy, and because of this the agenda setter must give a larger share of the budget to the legislative majority in order to maintain power.

The results from the models, taken together, show that assumptions about how agenda setting power carries over from one period to the next matter. The results provide insights into how to incorporate the Baron and Ferejohn (1989) legislative bargaining framework into a repeated environment. The most common assumption, that agenda setting power is randomly determined in each period, is not only the least realistic of the models, it exaggerates agenda setting power and produces the most pessimistic results about the wellbeing of the legislative majority. Assuming instead that a successful agenda setter automatically holds onto power keeps the framework just as tractable, but produces more attractive results from both a positive and normative perspective. That is, the model results in low turnover of agenda setter power, and predicts a more equitable division of the budget between the agenda setter and the legislative majority. What's more, we show that this model predicts the same equilibrium allocation in each period as the one-time equilibrium allocation in the non-repeated game. Alternatively assuming that an agenda setter must maintain the support of a legislative majority is the most realistic framework, and when legislators care

enough about future periods, it predicts low turnover of agenda setter power and produces the most attractive results about the wellbeing of the legislative majority.

Given the results, we see little reason to rely on the standard assumption that a new agenda setter is randomly selected each period when incorporating legislative bargaining into a repeated environment. Assuming that agenda setting power is exogenously determined by a random process (independent of whether the agenda setter successfully passes a proposal or has the support of the legislative majority) is both unrealistic and produces pessimistic results. Either of the alternative models has its attractive features. Our first model is easy to use and predicts a similar outcome as a non-repeated game. Our second model is more complicated, but is also the most realistic. Since the second model predicts a more equitable division of the budget, the results from the first model—as well as the non-repeated game—may be seen as an upper bound for agenda setter rent extraction, and a lower bound for the share of the budget allocated to the legislative majority.

2 Literature

Romer and Rosenthal (1978) develop a model of legislative decision making in which a legislature must choose a budget. An agenda setter (henceforth “AS” or “she”) first proposes a budget; then the entire legislature votes on whether to implement the proposal or keep the status quo in place. In equilibrium, the AS proposes a budget that maximizes her own district’s net transfer, while giving the legislative majority just enough pork barrel transfers to make it indifferent between voting in favor of or against the proposal. The AS’s monopoly over proposal power can result in her being the only legislator that is made better off by new legislation. Other legislators are at best indifferent between the new budget and the status quo.

Baron and Ferejohn (1989)’s legislative bargaining model essentially incorporates a Rubinstein (1982) bargaining game into the Romer-Rosenthal model. The primary difference between the Baron-Ferejohn and Romer-Rosenthal frameworks is that in the Baron-Ferejohn model, the game does not end when a proposal fails to gain majority support. Rather, a new legislator may be selected to propose a new budget allocation. That is, a single legislator does not have a monopoly over proposal power in the game. If the AS fails to pass a proposal, she may be replaced with a new AS.³ In Baron-Ferejohn, the AS’s proposal gives a majority of legislators just enough to entice them to vote in favor of the proposal. Here, however, the legislative majority requires larger transfers of pork to support a proposal than they did in Romer and Rosenthal (1978). Although the AS still captures the largest share of the budget compared with any other legislator, her share is smaller than in the Romer-Rosenthal game. Furthermore, the majority of legislators are better off compared to the status quo policy.

The current paper applies the Baron and Ferejohn (1989) model in repeated games. As such, a version of Baron and Ferejohn (1989)’s classic model is presented in Section 3 of this paper, and serves as a non-repeated benchmark throughout the analysis.

³As in Binmore (1987)’s extension of Rubinstein bargaining, Baron and Ferejohn (1989) assumes that the AS is determined by a stochastic process.

Many articles extend Baron and Ferejohn (1989)’s theoretical analysis (e.g., Baron 1996, Banks and Duggan 2000, Merlo and Wilson 1995, Banks and Duggan 2006, Eraslan 2002, Gersbach 2004, Snyder et al. 2005, Breitmoser Forthcoming), or apply it in various settings (e.g., Merlo 1997, Chari et al. 1997, Morelli 1999, Jackson and Moselle 2002). Our review focuses on the subset of articles that apply legislative bargaining in a repeated setting. Baron (1996) considers a repeated application in which the status quo any period is the previous policy that the legislature implemented. In equilibrium, agenda setters strategically propose policies (and manipulate the status quo) in order to limit the feasible proposals available to other agenda setters in the future. The emphasis of the model is an endogenous status quo policy, with the default policy equal to the most-recent implemented proposal. Kalandrakis (2004), Kalandrakis (2009), and Duggan and Kalandrakis (2010) generalize Baron’s results, allowing for multidimensional policy spaces.⁴ Battaglini and Coate (2007, 2008) allow the legislature to choose policies that affect government spending, taxes, and debt, considering how these variables fluctuate over time. McKelvey and Riezman (1992) consider dynamic legislative bargaining when legislators must stand for reelection after each period and the probability of serving as the AS is increasing in one’s seniority. Diermeier and Fong (2009) develop an alternative model of legislative bargaining in which an agenda setter has monopoly power over proposals, the status quo is determined by the most-recently implemented proposal, and the legislative process repeats with positive probability.

Each of these repeated applications of legislative bargaining assume that the status quo policy evolves over time, determined by past-period bargaining outcomes. To focus on how the status quo evolves over time, the articles make the simplifying assumption that AS power is exogenous, independent of past policy outcomes. This is the case when an AS is randomly selected each period (e.g., Duggan and Kalandrakis 2010), or when the identity of future ASs is common knowledge (e.g., Diermeier and Fong 2009). The current paper takes the opposite approach from the existing literature, with our analysis focusing on the rules governing how AS power changes over time, and their affect on equilibrium outcomes. To isolate the effects of our assumptions concerning AS power, we simplify the other aspects of the problem by assuming a stable, exogenous status quo policy. We are aware of no other article that focuses on the AS-authority aspect of the repeated environment.

3 A Benchmark: Non-Repeated Legislative Bargaining

We begin the analysis with a standard, non-repeated legislative bargaining model. This framework will serve as a foundation for the repeated models in Section 4.

There are n identical legislative districts, each represented by a single legislator. The n -member legislature is responsible for splitting a budget between the n districts. The total size of the budget equals 1. The share of the budget allocated to district $i = 1, \dots, n$ is denoted a_i . Vector $a = (a_{rand}, \dots, a_n)$ refers to the “allocation,” where $a_i \in [0, 1]$ for all i and $\sum_{i=1}^n a_i \leq 1$.

⁴See also ? who develop a model of dynamic bargaining between coalitions which allows for fully transferable utility between agents.

At the beginning of the game, a legislator is selected to serve as the initial-period agenda setter (henceforth “AS” or “she”). In period 1, the initial AS proposes an allocation. Then, the $n-1$ other legislators simultaneously vote on the proposed allocation. If at least m of the other legislators vote in favor of the proposal, then the budget is divided according to the proposal and the game ends. We refer to m as the “majority” although any $m \in \{1, \dots, n-2\}$ is allowed. If fewer than m other legislators vote for the proposal, then the proposal fails, and a new AS is randomly selected to propose another allocation. The process continues until a proposal passes, at which point the game ends.

Legislators discount future periods at a factor $\delta \in (0, 1)$. If a proposal passes in period t giving legislator i allocation a_i , then i earns period- t utility of $u_i^t = a_i$, and period-1 utility $u_i^1 = \delta^{t-1}a_i$. Following the literature, each legislator is equally likely to be selected as the AS in any period, the AS in each period randomly chooses which other legislators to include in a majority coalition (i.e., minimum winning coalition), and an indifferent legislator votes in favor of a proposal.

Equilibrium

To deal with the multiplicity of subgame perfect equilibria that arise in infinite period environments, the literature focuses on symmetric, subgame perfect, pure strategy, stationary Markov perfect equilibria, referred to simply as stationary equilibria (e.g., Baron and Ferejohn 1989). We do the same here. In a stationary equilibrium, the same strategy profile is played in every identical subgame, and strategies cannot condition on past periods.

The AS and other legislators face the same incentives in all periods of the game. Therefore in the stationary equilibrium, if the game does not end before some arbitrary period t , the period- t AS makes the same proposal and the other legislators follow the same voting strategies as were played by their counterparts in all previous periods.

To begin, it should be clear that the AS in any period will never propose an allocation that offers a positive allocation to more than m other legislators. Furthermore, the budget constraint will bind in equilibrium with $\sum_{i=1}^n a_i = 1$. This means that the AS in any period will propose an allocation that gives m other randomly selected legislators a positive allocation, denoted $a_M > 0$. She will also assign $a_{AS} = 1 - ma_M$ to herself, and $a_i = 0$ to all others that are not members of the majority coalition. The first period AS chooses a proposal such that it passes in the first round. We now determine the equilibrium levels of a_M and a_{AS} .

In any period t , a legislator votes for a proposal assigning him an allocation of at least δv , where v is his expected value from the game continuing into the next period. Therefore, $a_M = \delta v$, and $a_{AS} = 1 - m\delta v$. This implies that $v = \frac{1}{n}(1 - m\delta v) + \frac{m}{n}\delta v$. Solving for v gives $v = \frac{1}{n}$.

The equilibrium solution to the non-repeated game serves as a benchmark for the rest of the analysis. In each period of the unique stationary equilibrium, legislators vote in favor of any proposal that offers them at least δ/n ; and the AS proposes $a_{AS} = 1 - m\delta/n$ for herself, $a_M = \delta/n$ for m randomly selected legislators, and $a_i = 0$ for all others. Such a proposal passes in the first period.

4 Repeated Legislative Bargaining

The previous section presents a standard, non-repeated Baron and Ferejohn (1989) bargaining game. Our primary contribution comes in this section, where we compare different models of repeated legislative bargaining. In each of our models, the legislature plays a repeated version of the bargaining game from Section 3. When a proposed allocation passes, the game starts over the next period. Our models differ in whether and how an AS can hold onto power from one round of bargaining to the next.

If the period t proposal passes, who serves as AS in period $t+1$? The literature typically assumes that a new legislator is randomly selected to serve as the AS in the next period. In addition to considering this possibility, we propose two alternative frameworks in which a successful AS may hold onto power. The first possibility is that a successful AS automatically continues as the next period AS. In this case, a sitting AS loses power only if she is unable to pass a proposal. The second possibility is that the legislature first votes on whether to pass a proposal, then votes on whether to keep or replace the AS. The AS in period t continues as the AS in period $t + 1$ if and only if the majority of other legislators vote on her behalf. These are models of endogenous agenda setting authority.

We refer to each stage game—i.e., the period immediately following the passage of a proposal through the period in which the next allocation is passed—as a “round” of bargaining. The bargaining game within each round is identical to the non-repeated bargain game from Section 3. In the period following the passage of a proposal, the bargaining process begins over, with the identity of the initial AS in the new round dependent on the model.

In each of these models, the stage game is identical to the non-repeated game from the previous section. The total size of the budget in each period equals 1. The share of the period t budget allocated to district $i = 1, \dots, n$ is denoted a_i^t , where $a^t = (a_{rand}^t, \dots, a_n^t)$. Legislator i 's period- t utility is simply $u_i^t(a^t) = a_i^t$. If a proposed allocation does not pass in any period, that period's allocation assigns zero to all legislative districts. Discount factor $\delta \in (0, 1)$ applies to periods within a round of bargaining, i.e., when a proposal *does not* pass. Discount factor $\gamma \in (0, \delta]$ applies between rounds of bargaining.

In each of the models, an AS's proposal must receive votes from m other legislators to pass. If a legislator requires a positive allocation in order to vote for a proposal (or vote in favor of the AS in the second endogenous model), then m other legislators will be included in a majority coalition (i.e., minimum winning coalition) by the AS. M^t denotes the majority coalition in period t , and a_M^t denotes the period- t allocation assigned to a member of M^t . The analysis makes the following assumptions about M^t .

A 1 *If a new AS is drawn at the end of period t , then each legislator serves as period $t + 1$ AS with probability $\frac{1}{n}$, and is included in M^{t+1} with probability $\frac{m}{n}$.*⁵

⁵When the period- t AS loses power, she is just as likely to serve as the next period's AS as any other legislator. This assumption is purely for analytical convenience, and the results will continue to hold if we assume that the AS cannot serve as AS in the period immediately after she loses power.

A 2 *In the models with endogenous AS authority, if all members of majority coalition M^t vote in favor of the AS in period t , then the AS includes the same subset of legislators in the next period majority coalition M^{t+1} .*

In any period, the AS may be indifferent about which legislators to include in M ; the assumptions add structure to this choice.⁶ A 1 is standard for the literature. A 2 applies only in the models where agenda setting authority is endogenous. It implicitly assumes that the AS can commit to keeping a legislator in the majority coalition as long as he continues to vote in favor of the AS's proposals. This increases the expected payoffs members of the majority coalition get from allowing the current AS to hold onto power; they are assured of being included in the majority coalition again the next period if they vote in her favor. We discuss alternative assumptions in Section 4.5.

Finally, we follow the literature and assume the following.

A 3 *A legislator who is indifferent between voting in favor of or against a proposal (or sitting AS) will choose the alternative that he would choose if he were certain to cast the deciding vote. If he is still indifferent, he votes in favor of the proposal (or sitting AS).*

This rules out equilibria in which, for example, no legislator ever votes in favor of a (potentially beneficial) proposal, and no legislator has an incentive to deviate because no single legislator can pass a proposal on his own.

Infinite period models suffer from multiplicity of subgame perfect equilibria. To reign in the number of equilibria, we follow the literature and look for symmetric, subgame perfect, pure-strategy stationary Markov perfect equilibria, which we refer to simply as *stationary equilibria*. The solution concept puts a number of restrictions on equilibria including symmetry and subgame perfection. Most importantly, it allows us to restrict our attention to pure-strategy, stationary strategies which require that a legislator plays the same strategy in every period of the game. Combined with symmetry, this requirement means that the sitting AS makes the same proposal and the other legislators follow the same voting rule in every period. This also implies that a proposal either always passes or is always rejected. Any equilibrium found when restricting attention to stationary strategies will also be an equilibrium if we allow for a less-restrictive strategy set (although fewer restrictions mean other equilibria may also exist).⁷

Before considering the three models separately, it is helpful to establish the following lemma. It rules out the existence of stationary equilibria in which the proposed allocation fails.

⁶Baron and Herron (2003) consider a finite-length repeated bargaining game in which the status quo evolves over time but agenda setting power is exogenous. They show that even in a simple dynamic environment, the game is “remarkably poorly-behaved due to the opportunity of agenda setters to choose among majorities.” By lending some structure to the AS's choice of a majority coalition, our assumptions alleviate many of these problems.

⁷Limiting the strategy set to the set of stationary strategies greatly simplifies—but does not weaken—the analysis. This approach allows us to find all stationary equilibria, and each of the equilibria will continue to be equilibria when a broader class of strategies are allowed (Fudenberg and Tirole (1991), Chapter 13). This is because a player that wants to play a certain strategy in one period will also want to play the same strategy in all similar periods in the future. When we consider a player's incentive to deviate from the equilibrium strategy, because all others are already playing stationary strategies, the player will prefer to deviate in the same way in all periods—that is, his preferred deviation will also be a symmetric strategy.

Lemma 1 *In all three of the repeated models, there does not exist a stationary equilibrium in which a proposed allocation does not pass.*

All proofs are provided in the appendix. We provide intuition in the body of the paper. Suppose instead that the game was in an equilibrium in which a proposal does not pass. Then, legislators benefit from passing any proposal that provides them with a positive allocation. The AS now has an incentive to propose a small allocation for a majority of legislators keeping the rest of the budget for her own district, since such a proposal will pass making both the AS and the majority of legislators better off than with their equilibrium payoffs of zero. Hence, a contradiction. The lemma allows the discussion to focus on equilibria in which proposed allocations pass.

4.1 Random selection of AS

Most of the literature assumes a random selection of the agenda setter each period, independent of past success. In each period, a legislator is randomly selected to serve as AS. After a proposal passes, another round of bargaining starts with a new legislator randomly selected to serve as the initial AS in the new game. Denote this random-selection game by Γ_{rand} .

In each period of the stationary equilibrium, the AS proposes $a_M \geq 0$ for a randomly selected set of m other legislators, a_{AS} for herself, and $a_i = 0$ for all other legislators. Other legislators vote in favor of a proposal if and only if their expected lifetime utility from doing so is at least as large as their expected lifetime utility from voting against the proposal. A legislator's benefit from voting in favor of the proposed period t allocation is strictly increasing in a_i^t . Therefore, if i votes in favor of a proposal that offers him a_i^t , he will also vote in favor of any proposal that offers more than a_i^t . This means that we can represent a non-AS legislator's voting strategy in any period by a cut-rule \bar{a} , where he votes in favor of a proposal if and only if it offers him at least \bar{a} . Because of the symmetric and stationary equilibrium requirements, \bar{a} must be the same for all legislators and in all periods. Therefore, the stationary equilibrium of the game can be fully represented by a description of a_M and \bar{a} .

Proposition 2 describes the unique stationary equilibrium of game Γ_{rand} . A formal derivation is provided in the appendix.

Proposition 2 *In each period of Γ_{rand} : Legislators vote in favor of any proposal that assigns them at least*

$$\bar{a}^* = \frac{1}{n} \frac{\delta - \gamma}{1 - \gamma}.$$

The AS proposes $a_{AS}^ = 1 - m\bar{a}^*$ and $a_M^* = \bar{a}^*$, which passes. If $\delta = \gamma$, then $\bar{a}^* = a_M^* = 0$ and $a_{AS}^* = 1$. Given A 1, A 2, and A 3, this is the unique stationary equilibrium of the game.*

Since AS power and M^t are exogenously determined, the ex ante expected per period payoff to each legislator equals $\frac{1}{n}$. This means each player expects a discounted lifetime payoff of $\frac{1}{n} \frac{1}{1-\gamma}$ starting in any period, and a legislator expects a future payoff of $\delta \frac{1}{n} \frac{1}{1-\gamma}$ if the current period proposal does not pass, and a future payoff of $\gamma \frac{1}{n} \frac{1}{1-\gamma}$ if the current period proposal does pass. The

benefit to future utility of not passing a proposal therefore equals \bar{a}^* , as derived above. To entice a legislator to vote in favor of a proposal, it must offer the majority large enough allocations to offset the costs associated with the increased delay from passing a proposal. That is, a legislator will vote in favor of a current period allocation that provides him with at least \bar{a}^* . The AS sets $a_M = \bar{a}^*$, offering just enough to offset the costs of delay.

When $\gamma = \delta$, there is no extra delay associated with passing a proposal, in which case legislators never have an incentive to vote against a proposal. In this case, $\bar{a}^* = 0$ and the stationary equilibrium involves the AS capturing the entire budget for her own district.

4.2 Successful AS stays in power

The first endogenous model is similar to the random-selection model with one important exception: if a proposal passes, the AS that made the successful proposal continues on as the AS in the initial period of the next round of bargaining. Here, an AS holds onto power as long as she continues proposing allocations that pass. This model is particularly interesting because, as we show below, it results in the same per-period allocation as in the non-repeated benchmark game. Denote this game in which a successful AS automatically retains power by Γ_{auto} .

In this section, if a proposal passes in period t , then a new bargaining game begins in period $t+1$ with the successful AS from the previous period serving as the initial AS in the new bargaining game. Here again we can fully represent an equilibrium by majority-coalition allocation a_M , and voting cut rule \bar{a} . Proposition 3 describes the stationary equilibrium of the game.

Proposition 3 *In each period of Γ_{auto} : Legislators vote in favor of any proposal that assigns them at least*

$$\bar{a}^* = \frac{\delta}{n}.$$

The AS proposes $a_{AS}^ = 1 - m\bar{a}^*$ and $a_M^* = \bar{a}^*$, which passes. Given A 1, A 2, and A 3, this is the unique stationary equilibrium of the game.*

In equilibrium, a majority member who votes in favor of the period t proposal expects to earn a_M in the current period, and in all future periods. That is, his expected lifetime utility is $a_M \frac{1}{1-\gamma}$. Alternatively, voting against the period- t proposal results in expected per period utility of $\frac{1}{n}$ starting in the next period, and expected lifetime utility of $\frac{\delta}{n} \frac{1}{1-\gamma}$. Therefore, a majority member votes in favor of the proposal each period, as long as $a_i \geq \bar{a} = \frac{\delta}{n}$. The AS recognizes this and proposes the lowest acceptable allocation in each period.

4.3 Legislature votes on whether AS stays in power

In the second endogenous model, the legislature first votes on whether to pass an allocation proposal, and then in a separate vote decides whether to keep or replace the sitting AS. There are separate votes on the proposal and on AS power. Here, replacing an AS does not require that the legislature

turn down the current-period proposal. Rather, the legislature can vote in favor of a proposal, and still vote to replace the AS.

Formally, in each period the game takes place in the following order:

1. The AS makes a proposal.
2. The other $n - 1$ legislators vote on the proposal, which is implemented if m vote in its favor.
3. If the proposal passes, then the $n - 1$ legislators vote whether to keep or replace the sitting AS. If m other legislators vote in favor of the sitting AS, then she holds onto power into the next period. If fewer than m legislators vote in favor of the AS, or if the allocation proposal does not pass during the first vote, then a new legislator is randomly selected to serve as AS in the next period.

Denote this game by Γ_{vote} .

If a proposed allocation does not pass, a new legislator is automatically selected to serve as the next period's AS. Only after a proposal passes does the legislator vote on whether to keep or replace the AS. This structure means that our three models differ only in their transition between one round of bargaining and the next. Within each round of bargaining, the three models are identical.⁸

Lemma 1 establishes that any stationary equilibria must involve the proposed allocation passing in each period. Furthermore, the solution concept requires that strategies be symmetric and stationary, which means that the outcome of each period will be the same; either the AS always holds onto power, or she always loses power. This means that any stationary equilibria is one of two possible types, which we define here. In each period of a *high-turnover equilibrium*, the legislature first passes the proposed allocation, then votes to replace the sitting AS. In each period of a *low-turnover equilibrium*, the legislature first passes the proposed allocation, then votes to keep the sitting AS.

In any equilibria, legislators employ cut rules in their voting strategies. A legislator votes in favor of a proposal that offers him a large-enough allocation, then votes to keep the AS if she passed a proposal that provides him with a high(er)-enough allocation. A legislator requires a higher allocation in order to vote in favor of the AS, then he does to simply vote in favor of the proposal. In equilibrium, a legislator votes against a proposal if he is offered too small of an allocation, he votes in favor of the proposal then against the AS if the proposal provides him with a moderate allocation, and he votes in favor of the proposal then in support of the AS if the proposal provides him with a high-enough allocation. This means that the AS faces a choice between maximizing current-period payoffs and holding onto AS power. Her first option is to propose an allocation that gives a majority of legislators just enough allocation that they vote in favor of the proposal; this maximizes her current-period rent extraction, but results in her losing power. Her second option is to propose an allocation that provides other legislators with a larger share of the budget, enough

⁸It would be straightforward to assume instead that the legislature votes on the AS at the end of every period. This will not change the equilibrium allocation, as Lemma 1 established that proposals always pass in equilibrium.

that they will also vote to keep the AS in power; this reduces the AS's ability to extract rent in the current period, but increases her expected payoff in future periods. The AS's preferred option depends on her discount factor. If she is sufficiently impatient, then the AS chooses to maximize her current-period payoffs at the cost of losing power at the end of the period. If she cares enough about future periods, then the AS is willing to share a larger portion of the budget with other legislators in order to keep her agenda setting authority.

There exists two possible stationary equilibria; one is a high-turnover equilibrium and the other a low-turnover equilibrium. The high-turnover equilibrium exists when legislators are sufficiently impatient. The low-turnover equilibrium exists when legislators care enough about future periods. As we show below, at least one of these equilibria always exists, and there are some parameter values over which both exist. We walk through the formal derivation of equilibria in the appendix. Proposition 4 describes the equilibrium in detail.

Proposition 4 *In Γ_{vote} ,*

- *A high-turnover equilibrium exists iff*

$$\gamma \leq \frac{(1-\delta)m}{n-1-m}.$$

In each period of the high-turnover equilibrium,

- *Legislators vote in favor of any proposal that offers them at least $\frac{1}{n} \frac{\delta-\gamma}{1-\gamma}$, and vote in favor of an AS that provided them with at least $\frac{1}{n}$.*
- *The AS proposes an allocation in which $a_M^* = \frac{1}{n} \frac{\delta-\gamma}{1-\gamma}$, and $a_{AS}^* = 1 - \frac{m}{n} \frac{\delta-\gamma}{1-\gamma}$.*
- *A majority of legislators vote in favor of the proposal, then vote to replace the AS.*

- *A low-turnover equilibrium exists iff*

$$\gamma \geq \frac{(1-\delta)m}{n-1-\delta m}.$$

In each period of the low-turnover equilibrium,

- *Legislators vote in favor of any proposal that offers them at least $\frac{\delta}{n}$, and vote in favor of an AS that provided them with at least $\frac{1}{n}$.*
- *The AS proposes an allocation in which $a_M^* = \frac{1}{n}$, and $a_{AS}^* = 1 - \frac{m}{n}$.*
- *A majority of legislators vote in favor of the proposal, then vote to keep the AS.*
- *Given A 1, A 2, and A 3, when $\gamma > \frac{(1-\delta)m}{n-1-m}$, the low-turnover equilibrium is the unique stationary equilibrium. When $\gamma < \frac{(1-\delta)m}{n-1-\delta m}$, the high-turnover equilibrium is the unique stationary equilibrium. Otherwise, both equilibria exist.*

When legislators care little-enough about future periods, their strategies ignore any concerns about long-run agenda setting authority. In this case, the legislators are influenced by the same incentives as in the random-selection model, which results in the high-turnover here being equal to the standard model allocation. The low-turnover allocation here resembles the allocation in Γ_{auto} , except that in this setting the AS has less bargaining power and must therefore share a larger portion of the budget to stay in power. When legislators care enough about future periods, which is a common assumption in application, only the low-turnover equilibrium exists. As δ approaches 1, for example, the low-turnover equilibrium is the only equilibrium regardless of γ . As δ approaches γ , on the other hand, the low-turnover equilibrium is the only equilibrium as long as $\gamma > \frac{m}{n-1}$.

4.4 Comparing the Models

Although all of the models are similar in that their equilibria involve the AS capturing a larger-than-average share of the budget each period, and never sharing any of the budget with more than m other legislators, there are substantial differences between the three models. First of all, the random-selection model is the only model in which there does not exist the possibility an AS holding onto power across many periods.⁹ The other models allow for this (realistic) possibility. Additionally, the discount factors play a different role in each of the three model's equilibrium allocation. In standard model and the high-turnover equilibrium of Γ_{vote} , it is the difference between the two discount factors δ and γ that affect the allocation. In Γ_{auto} , only the intra-period discount factor, δ , affects the equilibrium allocation since voting against a proposal prevents the game from proceeding into a new round of bargaining. In the low-turnover equilibrium of Γ_{vote} , neither discount factor affects the allocation since the game proceeds from one round to the next each period, regardless of whether the legislature keeps or replaces the sitting AS.

These differences in equilibrium outcomes imply differences in the models' predictions about the magnitude of the per-period allocations and the long term expected utility for the various legislators. We discuss these differences in detail below.

Per-period allocation

In the equilibrium of the non-repeated benchmark model, the AS proposes a division of the budget that gives a majority of other legislators a positive allocation, but keeps the largest share for herself. The proposal passes in the first period and the game ends. In our repeated models, the outcome in each period looks similar to the one-time outcome of the non-repeated game. That is, a proposal passes every period that assigns the largest share of the budget to the AS. The portion of the budget captured by the AS, however, depends on the model.

Proposition 5 compares the per-period allocation in each of the repeated models. We use Γ_{voteHT} and Γ_{voteLT} to denote Γ_{vote} when it results in a high-turnover equilibrium and a low-

⁹We mean no systematic way for an AS to hold onto power. Technically, there is a random chance that an AS holds onto power even when she is replaced each period, as each legislator has a $1/n$ chance of being selected AS in each period that a new AS is selected.

turnover equilibrium, respectively. Remember, when legislators care enough about future periods, only the low turnover equilibrium exists.

Proposition 5

$$a_{AS}^*(\Gamma_{voteLT}) < a_{AS}^*(\Gamma_{auto}) < a_{AS}^*(\Gamma_{rand}) = a_{AS}^*(\Gamma_{voteHT}), \text{ and}$$

$$a_M^*(\Gamma_{rand}) = a_M^*(\Gamma_{voteHT}) < a_M^*(\Gamma_{auto}) < a_M^*(\Gamma_{voteLT}).$$

From a welfare perspective, a social planner interested in a single period’s allocation may prefer the game that results in an AS allocation closest to $\frac{1}{m}$; it is this allocation that divides the budget equally between all members of the majority coalition and the AS.¹⁰ Although none of the models achieves this equitable division of the budget, the low-turnover allocation in Γ_{vote} comes closest, followed by the allocation in Γ_{auto} . The random-selection model, Γ_{rand} , results in the least equitable division.

We can also compare the per-period allocations in the repeated models to the one-time allocation in the non-repeated game from Section 3. Γ_{rand} , is a straightforward generalization of the non-repeated framework. If the legislators do not care about future rounds of bargaining (i.e., $\gamma \rightarrow 0$), then the model produces the same results as the non-repeated benchmark in Section 3. Since $\gamma > 0$, however, the per-period allocation in Γ_{rand} always results in a larger transfer to the AS and a smaller transfer to other legislators compared with the non-repeated benchmark. This is also true in the high-turnover equilibrium of Γ_{vote} , which produces the same per period allocation as the standard model. In the low-turnover equilibrium of Γ_{vote} , on the other hand, the AS is willing to give up enough of her potential current-period allocation to hold onto power. The AS not only wants to pass a proposal, she also wants to maintain enough support to hold onto power. The dynamic concerns result in the AS sharing a larger portion of the budget than she does in the non-repeated game. The period allocation is strictly higher for the majority coalition members, and strictly lower for the AS in the low-turnover equilibrium of Γ_{vote} compared to the benchmark.

The model where a successful AS automatically retains power provides the most interesting comparison between the per-period allocation in a repeated game and the one-time allocation in the non-repeated game. The comparison warrants its own proposition.

Proposition 6 *In the stationary equilibrium of Γ_{auto} , the per-period allocation is identical to the one time allocation in the benchmark non-repeated game.*

Γ_{auto} results in the same per period allocation as the non-repeated benchmark. But, where the benchmark model produces the allocation in a game that lasts only one period in equilibrium, the repeated model sustains the allocation in each period of a repeated framework. That is, the allocation is achieved over and over again. This is an attractive result. It suggests that the non-repeated legislative bargaining game, and the results that come out of such models, do not unravel

¹⁰Of course, the most-equitable division results when the budget is split evenly among all n legislators, not just the AS and the legislative majority. However, none of the models result in the AS sharing the budget with more than m other legislators.

when we allow for repeated interactions. In fact, depending on the specifics of the repeated process, the predictions of the repeated and non-repeated models may be identical.

Expected lifetime utility

The previous section compares the per period allocation in each of the games, showing that systems of endogenous AS power result in a more-equal division of the budget in each period compared to the model of exogenous AS power. This section accounts for turnover in AS authority and considers the expected long-run utility in each game.

From an ex ante perspective, before an AS is selected, all legislators expect a per-period allocation of $\frac{1}{n}$. This is the case in all of the models, and implies that before the game begins, legislators are indifferent between the three models. Once the game begins and the AS and majority coalition are determined, however, legislators are not indifferent between the models. A more interesting look at expected utility considers expected payoffs from the perspective of an arbitrary period t , once the AS and M^t are known.

Proposition 7 considers which model of AS authority is preferred by the sitting AS, current members of the majority coalition, and other legislators from the beginning of any period. Let \succeq_{AS} , \succeq_M , and \succeq_o denote the preference-relation over the three repeated games by the sitting AS, current majority members, and other legislators, respectively.

Proposition 7 *If γ is high enough that Γ_{vote} achieves a low-turnover equilibrium, then*

$$\begin{aligned}\Gamma_{auto} &\succ_{AS} \Gamma_{vote} \succeq_{AS} \Gamma_{rand} \\ \Gamma_{vote} &\succ_M \Gamma_{auto} \sim_M \Gamma_{rand} \\ \Gamma_{rand} &\succ_o \Gamma_{auto} \sim_o \Gamma_{vote}.\end{aligned}$$

If γ is low enough that Γ_{vote} achieves a high-turnover equilibrium, then

$$\begin{aligned}\Gamma_{auto} &\succ_{AS} \Gamma_{vote} \sim_{AS} \Gamma_{rand} \\ \Gamma_{auto} &\sim_M \Gamma_{vote} \sim_M \Gamma_{rand} \\ \Gamma_{vote} &\sim_o \Gamma_{rand} \succ_o \Gamma_{auto}.\end{aligned}$$

Corollary 8 *When legislators care enough about future periods, the majority of legislators prefer the legislature to vote separately on proposed allocations and AS power (i.e., the majority prefer Γ_{vote}).*

The AS always prefers Γ_{auto} to either of the other repeated frameworks, as the model allows an AS to remain in power indefinitely, and her cost of doing so is less than in Γ_{vote} . The majority of legislators, on the other hand, are indifferent between Γ_{auto} , where they are included in the majority coalition indefinitely, and Γ_{rand} , where a new AS and majority coalition are randomly selected each period. This is because the majority coalition is given just enough in the first endogenous model to make them indifferent between accepting and rejecting the proposal. When legislators are

sufficiently patient (i.e., large enough γ), the majority of legislators strictly prefer playing Γ_{vote} , where members of the majority coalition have more bargaining power in their interaction with the AS, than in either of the other games. Other legislators, who are not the AS nor members of the majority coalition, prefer any game that results in the turnover of AS power between periods. Of course, their preferences change once they become the AS or a member of M in a future period.

4.5 Alternative Assumptions

Assumptions A 1 and A 2 add structure to an AS's choice of M^t . Furthermore, we assume that all legislators share the same probability of being selected as an agenda setter in any period. Here, we discuss alternative assumptions.

The first assumption, A 1, requires that a new majority coalition is made up of randomly selected legislators. Alternatively, we could allow the AS to strategically select the majority coalition, given the past votes cast by legislators. In equilibrium, a rational AS believes that a legislator who voted against an equilibrium allocation a_M^* in the past will do so again in the future, and will not include such a legislator in her own majority coalition. This changes the equilibrium allocation in each of the models.

In the non-repeated framework, for example, voting in favor of the equilibrium budget allocation earns a majority member a_M . If a majority member votes against the proposal, he sends the game into another period of bargaining in which he has a $1/n$ chance of being selected as AS. If he is not selected as AS, he will never be included in a new majority coalition as the new AS will recognize that he voted against receiving a_M in the previous period. Therefore, voting against the equilibrium allocation earns a majority member expected payoff $\delta \frac{1}{n}(1 - ma_M)$. In equilibrium, $a_M^* = \delta \frac{1}{n}(1 - ma_M^*)$; that is, $a_M^* = \frac{\delta}{n+m\delta}$. This alternative rule about majority coalition formation takes some of the bargaining power away from the majority coalition members. The equilibrium payoff for a member of the majority coalition is lower under this alternative rule, compared to under assumption A 1.

The same is true in the repeated frameworks. Allowing the AS to exclude legislators from the majority coalition based on their past voting record takes bargaining power away from the other legislators and allows the AS to capture a larger share of the budget each period.

The second assumption, A 2, requires that an AS who stays in power keeps the same majority coalition as long as they keep voting in her favor. Alternatively, we can relax this assumption to allow the AS to include any m legislators in future majority coalitions. Under the alternative assumption, the equilibria found assumption A 2 will still exist, with the AS choosing to maintain the same majority coalition and the other legislators voting accordingly. However, there will also exist other equilibria in which the AS follows a different voting rule when determining the majority coalition. For example, there will exist an equilibrium in which the AS randomly chooses the majority coalition each period, and the other legislators vote accordingly. The existence of these other equilibria do not add much to the analysis, as the AS prefers the equilibria found under A 2, since it is in this equilibrium that the AS is able to maintain majority support at the lowest costs.

Finally, we assume that each legislator has an equal chance of being selected AS in any period that a new AS is chosen. The allocation that must be given to buy the support of a legislator is increasing in the legislator's probability of being selected as AS himself in a future period. If we alternatively assume that agents differ in their probabilities of being selected AS, then an AS will build a majority coalition with the m other legislators who require the lowest payments in exchange for support—that is, those legislators with the lowest probability of being selected AS in a future period. The main results from our analysis should still carry over in such a setting, although the allocation details will change.

5 Conclusion

A growing literature considers repeated variants of Baron and Ferejohn (1989)'s legislative bargaining framework. These other papers tend to give little consideration to the rules governing agenda setting authority. The majority of these articles assume that a new AS is randomly selected each period, even after the successful passage of a proposal. Others assume that the order that legislators serve as the AS is fixed and common knowledge. In both cases, AS authority is independent of past success or failure as an AS, which is inconsistent with the realities of the legislative process.

We propose two alternative assumptions about how agenda setting authority can carry over from one round of bargaining to the next. In both frameworks, a successful AS may be able to continue on as AS the next period. The two models of endogenous proposal power each have attractive features. In contrast to the standard assumption about random power, these models predict low turnover of agenda setting power. Additionally, the models result in more attractive predictions concerning the AS's ability to extract rent from the legislative process, and the wellbeing of the legislative majority. Assuming that a successful AS automatically continues on as the AS in the next period is straightforward to incorporate into the model and solve, and predicts a similar outcome as in the non-repeated game. Assuming that a successful AS must maintain the support of a legislative majority to hold onto power is more realistic, and produces the most attractive results regarding the wellbeing of the legislative majority and limits to the AS's ability to extract rent from the legislative process.

Except for our assumptions governing agenda setting authority, the article assumes a basic structure for the bargaining environment. For example, we assume symmetric legislators, and an exogenous default allocation in the event that a proposal does not pass. These assumptions simplify the analysis and focuses the results on the rules governing agenda setting power. We discuss the relaxation of some assumptions in Section 4.5. However, allowing both the status quo and agenda setter identity to depend on past proposals is beyond the scope of this project, and is reserved for future work.

6 Appendix

Proof to Lemma 1

In a stationary equilibrium, symmetry implies that all legislators propose the same allocation when they serve as AS, and the proposal is the same in every period. Also, legislators employ the same voting strategy in every period, so either the proposed allocation will always pass or it will always fail. Suppose the equilibrium proposal fails. Then the proposal will fail in all periods, and the expected utility from voting against a proposal is zero. The expected utility from voting in favor of a proposal is at least zero. If a favorable vote results in strictly positive expected payoffs, then a legislator will deviate from their required strategy. If a favorable vote results in expected utility of zero, then the player is indifferent between voting in favor or against the proposal, and A 3 assures that such a legislator will vote for the proposal. Therefore, players always deviate from the equilibrium strategy of voting against the proposal; a contradiction. QED

Solving Γ_{rand} (Proof to Prop. 2)

The stationary equilibrium solution concept allows us to limit our attention to symmetric, stationary, pure strategies. This means that in any equilibrium, the equilibrium outcome is the same in each period of the game. All ASs make the same proposal and that proposal either passes in each period, or it fails each period. Lemma 1 rules out an equilibrium in which the proposal always fails.

We now evaluate the remaining possibility that a stationary equilibrium exists in which a proposal passes each period. A description of equilibrium must describe the allocation proposed by the sitting AS in each period, and the voting rule that other legislators follow when deciding whether to vote in favor of the proposal. It is straightforward to derive a general form of the voting rule given subgame perfection and A 3. That is, a legislator must vote against a proposal if and only if its passage decreases his expected discounted lifetime utility. He votes in favor of a proposed allocation that provides him a_i^t if

$$a_i^t + \gamma v_E \geq \delta v_D,$$

where v_E denotes the expected future equilibrium payoffs starting in the next period, and v_D denotes expected future payoffs if the legislator deviates from the equilibrium strategy to vote against the proposal. Remember, δ is the discount factor that applies within a round of bargaining, and $\gamma \leq \delta$ is the discount factor that applies between rounds. Thus, a legislator votes in favor of any proposal offering him at least

$$\bar{a}^* = \delta v_D - \gamma v_E,$$

where the cut value \bar{a}^* fully describes a legislator's equilibrium voting strategy in each period.

The sitting AS in each period t proposes a_M^t and a_{AS}^t . In equilibrium, the AS's proposal must maximize her expected utility, $a_{AS}^t + \gamma v_E$, subject to budget and incentive compatibility (IC) constraints. First, the proposal must satisfy the budget constraint, $a_{AS}^t + m a_M^t \leq 1$. Second, given that the proposal passes each period, it must satisfy an IC constraint for at least m other legislators.

This requires that $a_M^t \geq \bar{a}^*$. It is straightforward to show that both constraints bind with equality. If the budget constraint does not bind, then the AS can increase a_{AS}^t while keeping a_M^t constant. If the IC does not bind, then the AS can decrease a_M in order to increase a_{AS}^t . By the same logic, one can rule out the possibility that the proposal offers a positive allocation to more than m other legislators. Therefore, in equilibrium $a_M^t = \bar{a} = \delta v_D - \gamma v_E$ for all t . Thus, in each period the AS chooses a_M^t and a_{AS}^t such that $a_{AS}^t + ma_M^t = 1$ and $a_M^t = \bar{a}^*$.

In the stationary equilibrium, a_M^t and a_{AS}^t are the same across all periods, and can be denoted simply as a_M and a_{AS} . Thus, in equilibrium,

$$a_{AS} + ma_M = 1, \text{ and} \quad (1)$$

$$a_M = \bar{a}^* = \delta v_D - \gamma v_E. \quad (2)$$

The variable v_E in this model denotes the equilibrium expected utility for any legislator, starting in any period, before that period's AS is determined. In each period, a legislator has a $1/n$ chance of being AS and earning $a_{AS} = 1 - ma_M$, and a m/n chance of being in the majority coalition and earning a_M . This is true in all periods, therefore,

$$v_E = \frac{1}{1-\gamma} \left(\frac{1}{n}(1 - ma_M) + \frac{m}{n}a_M \right) = \frac{1}{1-\gamma} \frac{1}{n}. \quad (3)$$

Variable v_D denotes the expected future payoff to a legislator who deviates from the equilibrium strategy and votes against a_M . Therefore, v_D solves

$$v_D = \left(\frac{1}{n}(1 - ma_M) + \frac{n-m}{n}\gamma v_D + \frac{m}{n}\delta v_D \right).$$

That is,

$$v_D = \frac{1 - ma_M}{(1-\gamma)n - (\delta - \gamma)m}. \quad (4)$$

Solving the four equations, Eq. 1 - 4, gives the equilibrium solution for the proposed allocation, which we denote a_M^* and a_{AS}^* . From these, we also find a closed form solution for \bar{a}^* . Because $a_{AS}^* > a_M^*$, the AS strictly prefers to give a_M to m others rather than announce a proposal that does not satisfy the cut rule \bar{a}^* .

In each period of the stationary equilibrium, a legislator votes in favor of any proposal that offers him at least

$$\bar{a}^* = \frac{1}{n} \frac{\delta - \gamma}{1 - \gamma}, \text{ and} \quad (5)$$

the AS proposes $a_M^* = \bar{a}^*$ and $a_{AS}^* = 1 - m\bar{a}^*$. Such a proposal passes.

Both existence and uniqueness of the stationary equilibrium follows from the above analysis. We derive an equilibrium, and rule out other possible equilibria.¹¹

¹¹The analysis above relies heavily on the requirements that equilibrium strategies be stationary, symmetric, and subgame perfect. Weakening any of these requirements will result in additional equilibria.

In each period of equilibrium, the majority of legislators are offered just enough to make them indifferent between accepting and rejecting the proposed allocation. In this model, offering them anything at all is only necessary if the within-round discount factor is greater than the between-round discount factor. This may or may not be a reasonable assumption, depending on the application. If $\delta = \gamma$, then legislators never have an incentive to vote against a proposal since they expect the same future payoff regardless of their action. If $\delta = \gamma$, then $a_M = 0$ and $a_{AS} = 1$. QED

Solving Γ_{auto} (Proof to Prop. 3)

The first part of the analysis follows the analysis for Γ_{rand} . We can rule out an equilibrium in which proposals fail. In any equilibrium in which proposals pass, a legislator votes in favor of any proposal that offers him $a_i^t \geq \bar{a}^* = \delta v_D - \gamma v_E$, and the AS chooses a_M^t and a_{AS}^t such that $a_{AS}^t + ma_M^t = 1$ and $a_M^t = \bar{a}^*$. In equilibrium, a_M and a_{AS} are independent of t . Eq. 1 and 2 continue to hold in Model 1; however, the values v_D and v_E differ from the standard model.

If a member of the majority coalition votes in favor of the period t proposal and the proposal passes, then he will be included in the majority coalition in period $t + 1$. This is assured by A 2, and means that for a member of the majority coalition,

$$v_E = \frac{1}{1 - \gamma} a_M. \quad (6)$$

If the legislator deviates from his equilibrium strategy and instead follows a strategy of rejecting allocations of a_M , then he earns expected future utility v_D which solves $v_D = (1/n)(1 - ma_M)/(1 - \gamma) + (m/n)\delta v_D$, or

$$v_D = (1 - ma_M) \frac{1}{1 - \gamma} \frac{1}{n - m\delta}. \quad (7)$$

Solving Eq. 1, 2, 6, and 7 gives the equilibrium solution for the proposed allocation, which we again denote a_M^* and a_{AS}^* . From these, we also find a closed form solution for \bar{a}^* . Because $a_{AS}^* > a_M^*$, the AS strictly prefers to give a_M to m others rather than announce a proposal that does not satisfy the cut rule \bar{a}^* .

In each period of the stationary equilibrium, a legislator votes in favor of any proposal that offers him at least

$$\bar{a}^* = \frac{\delta}{n}, \text{ and} \quad (8)$$

the AS proposes $a_M^* = \bar{a}^*$ and $a_{AS}^* = 1 - m\bar{a}^*$. Such a proposal passes.

As was the case in the standard model, both existence and uniqueness of the stationary equilibrium follows from the above analysis. We derive an equilibrium, and rule out other possible equilibria. QED

Solving Γ_{vote} (Proof to Prop. 4)

In the second endogenous model, the legislature first votes on whether to pass the proposed allocation, then it votes on whether to keep or replace the sitting AS. Section 4.3 establishes that any

stationary equilibria must be either a high-turnover equilibrium, or a low-turnover equilibrium. In each period of a high-turnover equilibrium, the legislature first passes the proposed allocation, then votes to replace the sitting AS. In each period of a low-turnover equilibrium, the legislature first passes the proposed allocation, then votes to replace the sitting AS.

In both high- and low-turnover equilibria, a majority of legislators must prefer to vote in favor of the proposal. Just as in the earlier models, this requires that a legislative majority prefer for the proposed allocation to pass, which requires that $a_i^t \geq \delta v_D - \gamma v_E$ for a majority of legislators, where the values v_D and v_E differ between the high- and low-turnover cases.

Consider first the possibility of a high-turnover equilibrium. In this case, the game is similar to Γ_{rand} , and the values of v_E and v_D are given by Eq. 3 and 4. This means that in any high-turnover equilibrium, $a_M \geq \frac{1}{n} \frac{\delta - \gamma}{1 - \gamma}$. Additionally, in a high-turnover equilibrium the majority of legislators must prefer to replace the sitting AS in each period. This requires that $a_M + \gamma v_E > a_M \frac{1}{1 - \gamma}$, where the left hand side is the expected payoff from replacing the AS after each period, and the right hand side is the expected payoff from the AS holding onto power.¹² This expression simplifies to $a_M \leq \frac{1}{n}$. Since $\frac{\delta - \gamma}{1 - \gamma} < 1$, we can combine the two constraints on a_M to give us the legislators' full IC constraint,

$$\frac{1}{n} \frac{\delta - \gamma}{1 - \gamma} \leq a_M < \frac{1}{n}. \quad (9)$$

The AS chooses a_M and a_{AS} to maximize her own expected utility, $a_{AS} + \gamma v_E$, subject to Eq. 9 and the budget constraint. Thus, the AS prefers to propose $a_M^{HT} = \frac{1}{n} \frac{\delta - \gamma}{1 - \gamma}$ to any other a_M that satisfies the legislator IC constraint. Furthermore, she strictly prefers this a_M^{HT} to any lower value, since a lower value results in no allocation and a loss of power. Finally, it remains to be shown that the AS prefers to propose a_M^{HT} to any $a_M \geq \frac{1}{n}$, which would allow her to hold onto power. This will be the case when $(1 - ma_M^{HT}) + \gamma v_E \geq (1 - m \frac{1}{n}) \frac{1}{1 - \gamma}$, where the left hand side represents the AS's future utility if she holds onto power by offering the minimum allocation to the majority coalition that allows her to do so. Solving this inequality for γ gives the AS IC constraint,

$$\gamma \leq \frac{(1 - \delta)m}{n - 1 - m}. \quad (10)$$

When Eq. 10 is met, there exists a high-turnover equilibrium, as defined in Prop. 4.

Consider now the possibility of a low-turnover equilibrium, in which the same AS keeps power each period. In this case, a legislator who votes in favor of a proposal providing him a_M expects to receive a_M in future periods. Thus, $v_E = a_M \frac{1}{1 - \gamma}$. Voting against the proposal results in future expected utility δv_D , where

$$v_D = \frac{1}{n} (1 - ma_M) \frac{1}{1 - \gamma} + \frac{m}{n} \delta v_D.$$

¹²This expression assumes that an AS who keeps power uses the same majority coalition as in the previous period (i.e., A 2), and that legislators treat their vote as the deciding vote (i.e., A 3).

Solving for v_D gives,

$$v_D = \frac{1}{n}(1 - ma_M) \frac{1}{1 - \gamma} \frac{1}{1 - \frac{m}{n}\delta}.$$

In a low-turnover equilibrium, a_M must be such that a majority of legislators vote in favor of the proposed allocation, which requires that $a_M \frac{1}{1 - \gamma} \geq \delta v_D$. It must also be the case that a majority of legislators vote in favor of the sitting AS. This requires that $a_M \frac{1}{1 - \gamma} \geq a_M + \gamma \hat{v}_D$, where \hat{v}_D is the expected future utility from voting against an AS that provided a_M . It follows that

$$\hat{v}_D = \frac{1}{n}(1 - ma_M) \frac{1}{1 - \gamma} + \frac{m}{n}(a_M + \gamma \hat{v}_D).$$

Solving for \hat{v}_D gives,

$$\hat{v}_D = \left(\frac{1}{n}(1 - ma_M) \frac{1}{1 - \gamma} + \frac{m}{n}a_M \right) \frac{1}{1 - \frac{m}{n}\gamma}.$$

Clearly, $v_D < \hat{v}_D$, which means that a majority that votes in favor of the AS will necessarily prefer to vote for the proposed allocation. This renders the requirement that a majority prefer to pass the proposal redundant. It is sufficient to require that the majority of legislators prefer to keep the AS in power, which is the case when $a_M \geq \hat{v}_D$; that is,

$$a_M \geq \frac{1}{n}. \quad (11)$$

The AS chooses a_M and a_{AS} to maximize her own expected utility, $a_{AS} + \gamma v_E$, subject to Eq. 10 and the budget constraint. Thus, the AS prefers to propose $a_M^{LT} = \frac{1}{n}$ to any other a_M that satisfies the legislator IC constraint. She must also prefer a_M^{LT} to any $a_M < \frac{1}{n}$. The preferred deviation would offer the lowest a_M such that the majority of legislators will vote in favor of the proposal. Such an a_M solves $a_M \frac{1}{1 - \gamma} = \delta v_D$; that is, $a_M = \frac{\delta}{n}$. The AS therefore must prefer proposing $a_M^{LT} = \frac{1}{n}$, which allows her to hold onto power, rather than proposing $a_M = \frac{\delta}{n}$, which provides a higher current-period payoff but results in her losing power at the end of the period. This is the case when $(1 - ma_M^{LT}) \frac{1}{1 - \gamma} \geq (1 - m \frac{\delta}{n}) + \gamma \tilde{v}_D$, where \tilde{v}_D is the AS's expected future payoff from deviating. It solves

$$\tilde{v}_D = \frac{1}{n}(1 - m \frac{\delta}{n} + \gamma \tilde{v}_D) + \frac{m}{n}a_M^{LT} \frac{1}{1 - \gamma}.$$

Plugging in for $a_M^{LT} = \frac{1}{n}$ and solving for \tilde{v}_D gives

$$\tilde{v}_D = \frac{(1 - \gamma)n + (1 - \delta)m + m\gamma\delta}{(1 - \gamma)(n - \gamma)n}.$$

AS incentive compatibility therefore simplifies to

$$\gamma \geq \frac{(1 - \delta)m}{n - 1 - \delta m}. \quad (12)$$

When Eq. 12 is met, there exists a low-turnover equilibrium as defined in Prop. 4.

Notice that the right hand side of Eq. 12 is less than in Eq. 10, implying that there are values

of γ for which both high-turnover and low-turnover equilibria exists. The above analysis shows existence (i.e., at least one of the equilibria always exists), and, when it is the case, uniqueness of the stationary equilibria (i.e., when either Eq. 12 or Eq. 10, but not both, only one equilibrium exists). This aspect of the results relies on the assumptions made in the analysis, particularly A 2 and A 3, and on the assumptions implied by the solution concept, particularly symmetry and stationarity.

Related to this, the careful reader will notice that the above analysis relies on the fact that each player's potential strategy set is restricted to the set of stationary strategies. This means that if an AS proposes a_M this period, she is expected to continue to propose a_M in each period so long as she holds onto power. Furthermore, a legislator that votes against an offer of a_M this period will vote against similar offers in the future. Limiting the strategy set to the set of stationary strategies greatly simplifies the analysis, but it does not weaken the analysis. This approach allows us to find all stationary equilibria, and each of the equilibria will continue to be equilibria when a broader class of strategies are allowed (Fudenberg and Tirole (1991), Chapter 13). This is because a player that wants to play a certain strategy in one period will also want to play the same strategy in all similar periods in the future. When we consider a player's incentive to deviate from the equilibrium strategy, because all others are already playing stationary strategies, the player will prefer to deviate in the same way in all periods—that is, his preferred deviation will also be a symmetric strategy. QED

Proof to Prop. 5

Follows immediately from a comparison of a_{AS}^* and a_M^* as established by the three models. QED

Proof to Prop. 6

Follows immediately from a comparison of a_{AS}^* and a_M^* from Model 1, with the allocation in the benchmark equilibrium. QED

Proof to Prop. 7

From the perspective of any period t , U_{AS} denotes the expected lifetime utility for the sitting AS, U_M denotes the expected lifetime utility of any member of M^t , and U_o denotes the expected lifetime utility for any other $i \notin \{AS, M^t\}$.

For Γ_{rand} , and for the high-turnover equilibrium of Γ_{vote} ,

$$\begin{aligned} U_{AS}(\Gamma_{rand}) &= U_{AS}(\Gamma_{voteHT}) &= 1 - \frac{m}{n} \frac{\delta - \gamma}{1 - \gamma} + \frac{1}{n} \frac{\gamma}{1 - \gamma} \\ U_M(\Gamma_{rand}) &= U_M(\Gamma_{voteHT}) &= \frac{1}{n} \frac{\delta - \gamma}{1 - \gamma} + \frac{1}{n} \frac{\gamma}{1 - \gamma} = \frac{\delta}{n} \frac{1}{1 - \gamma} \\ U_o(\Gamma_{rand}) &= U_o(\Gamma_{voteHT}) &= \frac{1}{n} \frac{\gamma}{1 - \gamma}. \end{aligned}$$

For Γ_{auto} ,

$$\begin{aligned} U_{AS}(\Gamma_{auto}) &= (1 - m\frac{\delta}{n})\frac{1}{1-\gamma} \\ U_M(\Gamma_{auto}) &= \frac{\delta}{n}\frac{1}{1-\gamma} \\ U_o(\Gamma_{auto}) &= 0. \end{aligned}$$

For the low-turnover equilibrium of Γ_{vote} ,

$$\begin{aligned} U_{AS}(\Gamma_{voteLT}) &= (1 - \frac{m}{n})\frac{1}{1-\gamma} \\ U_M(\Gamma_{voteLT}) &= \frac{1}{n}\frac{1}{1-\gamma} \\ U_o(\Gamma_{voteLT}) &= 0. \end{aligned}$$

It is straight forward to show that

$$\begin{aligned} U_{AS}(\Gamma_{voteHT}) &= U_{AS}(\Gamma_{rand}) \leq U_{AS}(\Gamma_{voteLH}) < U_{AS}(\Gamma_{auto}) \\ U_M(\Gamma_{voteHT}) &= U_M(\Gamma_{rand}) = U_M(\Gamma_{auto}) < U_M(\Gamma_{voteLH}) \\ U_o(\Gamma_{voteLH}) &= U_o(\Gamma_{auto}) < U_o(\Gamma_{voteHT}) = U_o(\Gamma_{rand}). \end{aligned}$$

The preference relations stated in the proposition follow immediately from these inequalities. QED

Proof to Cor. 8

When $\gamma \geq \frac{(1-\delta)m}{n-1-m}$, the low-turnover equilibrium is the unique equilibrium of Γ_{vote} . As $\delta \rightarrow 1$, the right hand side of this inequality approaches 0. Therefore, for any $\gamma > 0$, there exists a value of $\delta < 1$ such that the low-turnover equilibrium is unique. The same will be true as $\gamma \rightarrow 1$, as long as the assumption that $\gamma \leq \delta$ is maintained. Prop. 7 already established that when Γ_{vote} achieves a low-turnover equilibrium, the majority of legislators prefers Γ_{vote} to either of the other models. QED

References

- Banks, Jeffrey S. and John Duggan**, “A Bargaining Model of Collective Choice,” *American Political Science Review*, 2000, 94 (1), 73–88.
- **and** –, “A General Bargaining Model of Legislative Policy-Making,” *Quarterly Journal of Political Science*, 2006, 1, 49–85.
- Baron, David P.**, “A Dynamic Theory of Collective Goods Programs,” *American Political Science Review*, 1996, 90 (2), 316–330.
- **and John A. Ferejohn**, “Bargaining in Legislatures,” *American Political Science Review*, 1989, 83 (4), 1181–1206.

- **and Michael Herron**, “A Dynamic Model of Multidimensional Collective Choice,” in K Kollman, J Miller, and S Page, eds., *Computational Models in Political Economy*, Cambridge, MA: MIT Press, 2003.
- Battaglini, Marco and Stephen Coate**, “Inefficiency in Legislative Policymaking: A Dynamic Analysis,” *American Economic Review*, 2007, *97* (1), 118–149.
- **and –**, “A Dynamic Theory of Public Spending, Taxation, and Debt,” *American Economic Review*, 2008, *98* (1), 201–236.
- Binmore, Kenneth**, “Perfect Equilibria in Bargaining Models,” in K Binmore and P Dasgupta, eds., *The Economics of Bargaining*, New York: Basil Blackwell, 1987.
- Breitmoser, Yves**, “Parliamentary bargaining with priority recognition for committee members,” *Social Choice and Welfare*, Forthcoming. DOI: 10.1007/s00355-010-0486-1.
- Chari, V. V., Larry E. Jones, and Ramon Marimon**, “The Economics of Split-Ticket Voting in Representative Democracies,” *American Economic Review*, 1997, *87* (5), 957–976.
- Diermeier, Daniel and Pohan Fong**, “Legislative Bargaining with Reconsideration,” 2009. working paper.
- Duggan, John and Tasos Kalandrakis**, “Dynamic Legislative Policy Making,” 2010. working paper, University of Rochester.
- Eraslan, Hulya K. K.**, “Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model,” *Journal of Economic Theory*, 2002, *103*, 11–30.
- Fudenberg, Drew and Jean Tirole**, *Game Theory*, Cambridge, MA: The MIT Press, 1991.
- Gersbach, Hans**, “Dividing resources by flexible majority rules,” *Social Choice and Welfare*, 2004, *23* (2), 295–308.
- Jackson, Matthew O. and Boaz Moselle**, “Coalition and Party Formation in a Legislative Voting Game,” *Journal of Economic Theory*, 2002, *103* (1), 49–87.
- Kalandrakis, Tasos**, “A Three-Player Dynamic Majoritarian Bargaining Game,” *Journal of Economic Theory*, 2004, *116*, 294–322.
- , “Minimum Winning Coalitions With Endogenous Status Quo,” *International Journal of Game Theory*, 2009. DOI: 10.1007/s00182-009-0202-5.
- McKelvey, Richard and Raymond Riezman**, “Seniority in Legislatures,” *American Journal of Political Science*, 1992, *86* (4), 951–965.
- Merlo, Antonio**, “Bargaining Over Governments in a Stochastic Environment,” *Journal of Political Economy*, 1997, *105* (1), 101–131.

- **and Charles A. Wilson**, “A Stochastic Model of Sequential Bargaining with Complete Information,” *Econometrica*, 1995, *35* (2), 371–399.
- Morelli, Massimo**, “Demand Competition and Policy Compromise in Legislative Bargaining,” *American Political Science Review*, 1999, *93* (4), 809–820.
- Romer, Thomas and Howard Rosenthal**, “Political Resource Allocation, Controlled Agendas, and the Status Quo,” *Public Choice*, 1978, *33*, 27–43.
- Rubinstein, Ariel**, “Perfect Equilibrium in a Bargaining Model,” *Econometrica*, 1982, *50* (1), 97–110.
- Snyder, James M., Michael M. Ting, and Stephen Ansolabehere**, “Legislative Bargaining under Weighted Voting,” *American Economic Review*, 2005, *95* (4), 981–1004.