Loss Aversion, Survival and Asset Prices

David Easley and Liyan Yang*

Abstract

This paper studies the wealth and pricing implications of loss aversion in the presence of arbitrageurs with Epstein-Zin preferences. Our analysis shows that if loss aversion is the only difference in investors’ preferences, then for empirically relevant parameter values, loss-averse investors will be driven out of the market and do not affect long run prices. The selection process is slow in terms of wealth shares; but it is effective in terms of price impacts, because of endogenous withdrawal by loss-averse investors from the stock market. Overall, the market selection mechanism is efficient.

Key words: loss aversion; Epstein-Zin preferences; market selection; asset pricing

JEL Classification Numbers: G12, D50

*Easley, dae3@cornell.edu, Department of Economics, Cornell University, Ithaca, NY 14853. Yang, liyan.yang@rotman.utoronto.ca, Department of Finance, Joseph L. Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario M5S 3E6. We thank Nick Barberis, Larry Blume, Melanie Cao, Ming Huang, Mark Huson, Peng Lin, Peter Liu, Guo Ying (Rosemary) Luo, Randall Morck, Ted O’Donoghue, Maureen O’Hara, Cedric Okou, Hersh Shefrin, Viktor Tsyrennikov, Kevin Wang, Jason Wei, Xiong Wei, Hongjun Yan, and the audiences at 2011 Queens Behavioral Economics Conference (Kingston, Canada), 2011 UCLA Behavioral Finance Conference (Los Angeles, USA), Financial Economics Workshop of University of Toronto, and Finance Seminars at Alberta School of Business, DeGroote School of Business at McMaster University, Guanghua School of Management at Peiking University, and Schulich School of Business at York University for helpful comments. All errors are our responsibility and we welcome any comments.
1 Introduction

The behavior of individuals in experiments is sometimes inconsistent with those individuals being well described as expected utility maximizers with correct expectations. Similarly, aggregate outcomes in asset markets are sometimes inconsistent with the predictions derived from (correct) expected utility maximizers interacting in a well functioning market.\footnote{See Barberis and Thaler (2003) for a survey on the extensive evidence.} These observations have motivated the study of various alternative decision theories. One particularly interesting alternative theory studied in the recent behavioral finance literature is loss aversion which is a salient feature of prospect theory. Researchers have found that loss aversion helps to explain many financial phenomena, including the high mean, excess volatility and predictability of stock returns (e.g., Barberis, Huang and Santos, 2001); the value effect (Barberis and Huang, 2001); and, the GARCH effect in stock returns (McQueen and Vorkink, 2004).

Studies of the impact of loss aversion on markets are typically conducted in representative agent frameworks in which there is only one investor or equivalently all investors are identical.\footnote{E.g., Benartzi and Thaler (1995). Barberis and Huang (2001, 2007, 2009), Barberis, Huang and Santos (2001), McQueen and Vorkink (2004), Grünea and Semmler (2008).} This research has produced valuable insights into the potential for loss aversion to explain asset market puzzles, but it has a serious limitation. In particular, there is no trade, as there is no one to trade with in the economies studied in this literature. It’s not just the absence of trade that is troubling, rather it’s whether the trade that would occur between heterogeneous individuals would dampen or even eliminate the impact of loss aversion on market outcomes, such that financial markets can be understood, to a large extent, by models with traditional investors who are not loss-averse. This concern led Barberis and Huang (2009, p 1567) to caution that one should interpret the equity premium obtained in their representative agent model as “an upper bound on the equity premium that we would obtain in a more realistic heterogeneous agent economy.”

Formally, the following questions are left unanswered in the literature: In a heterogeneous agent economy, do loss-averse investors gradually lose their wealth share to those investors who are not loss-averse? Will loss-averse investors eventually be driven out of the market in the sense that their wealth share converges to zero? If so, how quickly? Are the implications...
for wealth and price impact of loss aversion the same? Say, can loss-averse investors have a long-run impact on asset prices even when their wealth share eventually becomes negligible? Do the wealth share and price impact converge at the same speed in the selection process? To answer these questions regarding the long-run and short-run wealth and pricing implications of loss aversion, some formal modeling is needed.

In this paper, we take up this task, and study the question of whether and how loss-averse investors can survive and influence prices. We analyze a heterogeneous agent economy with two (classes of) investors and two tradable assets—a risk-free bond and a risky stock. Both investors have recursive preferences. The first investor, labeled the EZ-investor, has Epstein-Zin preferences (Epstein and Zin, 1989) and she represents “rational investors” or “arbitrageurs”. We use Epstein-Zin preferences in order to disentangle risk aversion parameter and elasticity of intertemporal substitution parameter (EIS henceforth) and also to make this rational investor directly comparable with our second, loss-averse investor. The second investor is called the LA-investor, and he has a recursive preference representation proposed by Barberis and Huang (2007, 2009).\(^3\) The LA-investor departs from the EZ-investor in the way he evaluates his investment in the stock market: he derives utility from investing in the market both indirectly, via its contribution to his lifetime consumption, and directly, via its resulting fluctuations in his financial wealth, and he is more sensitive to losses than to gains (loss aversion).

We have two sets of results which, respectively, concern the long-run and short-run wealth and pricing implications of loss aversion, and both sets of results suggest that the market selection mechanism is effective. First, if investors only differ in whether they are loss-averse or not, the LA-investor will be driven out of the market and will have no impact on asset prices in the long run for economies with empirically relevant parameter values. This result is driven primarily by the endogenous difference in investors’ equilibrium portfolio choices. Previous studies on portfolio selection among expected utility maximizers show that the closer an investor’s utility is to log utility, the higher is his/her expected wealth growth rate (De Long, Shleifer, Summers and Waldman, 1991; Blume and Easley, 1992). Our analysis shows that this insight holds for recursive preferences in a general equilibrium

\(^3\)Throughout this paper, we will use “she”/“her” to refer to the EZ-investor and use “he”/“him” to refer to the LA-investor.
setting. Under empirically plausible parameter values, the EZ-investor is more risk averse than the log utility, but the nature of loss aversion makes the LA-investor act as if he is even more risk averse than the EZ-investor, and therefore further from the log utility investor. Thus, the LA-investor vanishes.\footnote{Although the intuition for this result comes from the previous literature, the analysis is nonetheless complex because of the dynamic portfolio choice and savings decisions that our investors face. In particular, the result does not follow immediately from the previous literature as loss aversion also causes the LA-investor’s saving behavior to be endogenously different from the EZ-investor’s, which might affect the LA-investor’s survival prospects. We nonetheless demonstrate that this loss-aversion-induced difference in savings cannot overcome the LA-investor’s disadvantage from his portfolio choice.}

We also find that EIS and time-patience parameters determining intertemporal behavior matter dramatically for survival. Small differences in these parameters can easily offset the negative effects of loss aversion. For instance, in a calibrated economy, a difference in the (annualized) time-patience parameter of two percent can result in the long run dominance of the LA-investor. This result suggests that what matters for the long-run survival is whether investors optimize over savings decisions.\footnote{Yan (2008) also makes a similar statement in an economy populated by investors with constant relative risk aversion (CRRA) preferences. Our analysis sharpens Yan’s statement because the recursive preferences in our model separate EIS and risk aversion parameters.} So, whether loss-averse investors survive crucially depend on whether they have a high enough saving motive. However, Tanaka, Camerer and Nguyen (2010) conducted experiments in Vietnamese villages and found that loss-averse investors are also relatively impatient, which therefore, according to our analysis, suggests that loss-averse investors are likely to lose out in the long run.

Our second set of results address the timespan of the selection process, which further quantifies the strength of the market selection mechanism. The most striking result emerging from this analysis is that the wealth dynamics and the price dynamics are significantly different. The selection process is slow in terms of wealth shares. For example, in calibrated economies, after 50 years, the LA-investor, on average, retains more than 70\% of his initial wealth share. However, the selection mechanism is effective in terms of price impacts for two reasons. First, introducing non-loss-averse investors can immediately reduce the price impact of loss aversion. The equity premium in a heterogeneous agent economy with the LA-investor and the EZ-investor each controlling half the aggregate wealth is much smaller than that in a representative agent economy populated only with the LA-investor, and this difference can be as large as ten times in calibrated economies. Second, even if the LA-investor starts
with a very high wealth share (say, greater than 90%) in the heterogeneous agent economy, so that the initial equity premium is close to the one generated by the representative loss-averse agent model, the price impact of loss aversion drops quickly in the heterogeneous agent economy—after 50 years, the equity premium, on average, retains about 30% of its initial value in calibrated economies.

The effectiveness of the selection force in reducing the price impact of loss aversion is due to its first-order risk aversion feature. Specifically, loss aversion means that the investor’s utility function has a kink at a reference point, which represents the first-order risk aversion. It is this kink that raises the equity premium in the single representative agent model, because first-order risk aversion implies that the risk premium is proportional to the standard deviation of returns, as opposed to the expected utility preference (known as second-order risk aversion) where the risk premium is proportional to the variance of returns (Segal and Spivak, 1990). However, in a heterogeneous agent economy, this kink also causes the LA-investor optimally chooses not to purchase the risky asset, leaving the equity premium to be largely determined by the EZ-investor. So, interestingly and surprisingly, it is the same kink feature of loss aversion that is once desired to generate a sizable equity premium in the representative agent economy and now becomes a limitation for loss aversion to exert a large price impact in the heterogeneous agent economy.

The result that wealth and price impacts have different selection speeds in our model is in sharp contrast to those obtained in the recent selection models using traditional CRRA preferences (e.g., Yan 2008; Dumas, Kurshev and Uppal, 2009; Fedyk, Heyerdahl-Larsen and Walden, 2012). In those alternative models, wealth and price impacts always move in the same order. For example, Yan (2008) shows that the selection process is excessively slow both in terms of wealth and in terms of price impact in an economy where CRRA investors disagree about the risk of a risky asset; Fedyk, Heyerdahl-Larsen and Walden (2012) argue that the selection speed can become quick once investors disagree about multiple risks. The different wealth and price impact dynamics in our model illustrates that loss aversion is distinct from the traditional smooth risk aversion in driving aggregate outcomes in financial

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6 This result also echoes Kogan, Ross, Wang and Westerfield (2006, 2011) and Cvitanič and Malamud (2011) who underscore that long-run survival and price impact are two distinct concepts. Our result complements theirs by highlighting the distinction in the short-run.
markets.

The remainder of this section reviews the relevant literature. Section 2 outlines the model, and Section 3 characterizes the equilibrium. Section 4 demonstrates the implications for survival and price impact of loss aversion when it is the only difference in investors’ preferences. Section 5 discusses its implications for survival when investors also have different EIS parameters or time-patience parameters. Section 6 concludes. The appendix provides the first-order conditions characterizing investors’ decisions for the case of unit EIS and the details of the numerical algorithm.

1.1 Literature Review

This paper contributes to two strands of literature. The first is the market selection literature, which studies what types of investors survive and have a price impact in a dynamic economy. So far, this literature has primarily focused on selection over beliefs and not over preferences. Although the idea of market selection dates back to the early 1950s (Alchian, 1950; Friedman, 1953), rigorous analysis of this idea has only recently been done. De Long, Shleifer, Summers, and Waldman (1991) are the first who cast doubts on the idea of market selection. They rely on partial equilibrium analysis and show that investors with incorrect beliefs can survive. Blume and Easley (1992) show that incorrect beliefs can be an advantage for survival in models with endogenous asset prices but exogenous savings decisions. Sandroni (2000), Blume and Easley (2006) and Yan (2008) endogenize both savings and portfolio decisions and show that only investors with beliefs closest to the objective probabilities will survive in economies with bounded aggregate endowments. Kogan, Ross, Wang and Westerfield (2009) demonstrate that in economies with unbounded endowments, investors with incorrect beliefs may survive. Kogan, Ross, Wang and Westerfield (2006, 2009) and Cvitanić and Malamud (2011) point out that survival and price impact can be different concepts in the long run.

Investors in all of the above models have time-separable utility functions. Borovička (2012) has recently studied the belief-selection problem in an economy with Epstein-Zin preferences and found that agents with distorted beliefs are not driven out of the market.

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7 One exception is Condie (2008), which analyzes the market selection problem for an economy populated with ambiguity averse investors and expected utility investors.
for an empirically relevant range of parameters. Other studies on market selection consider issues related to incomplete markets (Coury and Sciubba, 2005; Sandroni, 2005; Blume and Easley, 2006; Gallmeyer and Hollifield, 2008; Cao, 2011), imperfect competition (Palomino, 1996; Kyle and Wang, 1997), comparison of trading rules (Blume and Easley, 1992; Amir, Evstigneev, Hens and Schenk-Hoppé, 2005; Böhm and Wenzelburger, 2005), and asymmetric information and learning (Mailath and Sandroni, 2003; Sciubba, 2005; Cogley and Sargent, 2009). Instead of studying belief selection, this paper analyzes preference selection in frictionless and complete market economies, and it is the first study on the market selection problem between loss aversion and Epstein-Zin preferences.

The second strand of related literature considers the role of loss aversion in determining trading behavior, asset prices and trading volumes. Loss aversion, that investors are more sensitive to reductions in the value of their financial wealth than to gains, is a key feature of prospect theory which was introduced by Kahneman and Tversky (1979). Berkelaar, Kouwenberg and Post (2004), Gomes (2005), Kyle, Ouyang and Xiong (2006), and Barberis and Xiong (2009) study the optimal portfolio choice problem under loss aversion. Benartzi and Thaler (1995) were the first to use loss aversion to explain the equity premium puzzle. Barberis, Huang and Santos (2001) extend Benartzi and Thaler’s setting to a dynamic model and find that combining loss aversion and the “house-money effect” helps to explain the behavior of the aggregate stock market. Barberis and Huang (2001) find that loss aversion is also useful in understanding the value effect in the cross-section of stock returns. Grünea and Semmler (2008) study a production economy and find that a model incorporating loss aversion can match data much better than pure consumption-based asset-pricing models. McQueen and Vorkink (2004) show that loss aversion helps to explain the asymmetric GARCH properties of stock returns. Barberis and Huang (2007, 2009) propose a preference specification that incorporates both loss aversion and narrow framing and study its applications in portfolio choice and asset pricing.

All of the above-mentioned asset-pricing models are conducted in a representative agent framework. Gomes (2005), Gabaix (2007) and Berkelaar and Kouwenberg (2009) explore

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8 There is a large literature examining the specific pricing implications of heterogeneous risk attitudes and beliefs, including Wang (1996), Basak and Cuoco (1998), Basak (2005), Bhamra and Uppal (2009), Basak and Yan (2010), Xiong and Yan (2010), and Yan (2010), among others.
the interaction between loss-averse investors and expected utility maximizers. However, all three studies have a finite horizon model and are therefore unable to answer the question of whether loss-averse investors survive and affect prices in the long run.

2 The Model

We analyze a pure exchange economy with one perishable consumption good, which is the numeraire. Time is discrete and lasts forever: $t = 0, 1, 2, \ldots$. There are two assets—a risk-free bond and a risky stock. The bond is in zero net supply and earns a gross interest rate of $R_{f,t}$ between time $t$ and $t + 1$. The stock is a claim to a stream of the consumption good represented by the dividend sequence $\{D_t\}_{t=0}^{\infty}$. It is in limited supply (normalized to 1) and is traded in a competitive market at the (ex-dividend) price $P_t$. Let $f_t \equiv \frac{P_t}{P_{t+1}}$ and 

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t}$$

be the price-dividend ratio at time $t$ and the gross return on the stock between time $t$ and $t + 1$, respectively.

The dividend growth rate $\theta_{t+1} \equiv \frac{D_{t+1}}{D_t}$ is i.i.d. over time and follows a distribution given by

$$\theta_{t+1} = \begin{cases} 
\theta_H, & \text{with probability } \pi_H, \\
\theta_L, & \text{with probability } \pi_L,
\end{cases}$$

(1)

with $0 < \theta_L < \theta_H$, $0 < \pi_H < 1$ and $\pi_L = 1 - \pi_H$. We use a binomial distribution for the dividend growth rate process so that the two tradable assets induce a dynamically complete financial market. This ensures that our results on survival are driven by the difference in investors’ preferences and not by any assumed incompleteness in the financial-market structure. The market structure is important as whether the market-selection argument is valid depends crucially on the completeness of financial markets (see, among others, Blume and Easley, 2006; Cao, 2011).

We follow the literature in assuming that the aggregate consumption and aggregate dividends are equal.\(^9\) Under this assumption, even a representative agent economy with loss-aversion preferences cannot match the historical equity premium,\(^{10}\) as the equilibrium stock

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\(^9\)For consumption-based models, see, among others, Lucas (1978) and Mehra and Prescott (1985); for models studying loss aversion, see, among others, Gomes (2005) and Berkelaar and Kouwenberg (2009).

\(^{10}\)See the first economy studied by Baberis, Huang and Santos (2001), and Subsection 4.1 below.
returns are not volatile enough to induce the loss-averse investor to abandon the stock market. We have extended our analysis to a three-asset setting which is capable of generating the historical equity premium via a combination of loss aversion and narrow framing, and have found that all our main results hold in this extended model. To focus on our selection results in the most transparent setting, this extension is not reported in the paper.

The economy is populated by two (classes of) investors, who are distinguished by their preferences. The first investor, labeled the *EZ-investor*, derives utility from intertemporal consumption plans according to Epstein-Zin preferences (Epstein and Zin, 1989). The second investor, labeled the *LA-investor*, is the investor emphasized in the behavioral finance literature, see Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), Barberis, Huang and Thaler (2006) and Barberis and Huang (2007, 2009). This investor gets utility not only from consumption but also from fluctuations in the value of his stock holdings, and he is loss-averse over these fluctuations.

We use the preference specification developed by Barberis and Huang (2007, 2009) to describe the LA-investor’s preferences. According to this specification, the EZ-investor’s preference is simply a degenerate case of the LA-investor’s preference, where the parameter controlling the term related to loss aversion is set to be zero. Thus, this preference specification allows us to isolate the impact of loss aversion on the LA-investor’s wealth dynamics (survival) and asset prices.

We choose Epstein-Zin preferences to represent arbitrageurs for two additional reasons. First, Epstein-Zin preferences allow us to separate the risk-aversion parameter and the EIS parameter. These two parameters presumably have very different roles in determining investors’ survival prospects, as the existing market-selection literature suggests that portfolio decisions, which are more related to risk aversion, and saving behaviors, which are more related to EIS, affect survival in different ways. Second, Epstein-Zin preferences deserve more serious investigation on their own, as the recent literature has shown that Epstein-Zin preferences help to explain many salient features of the financial market.$^{11}$

Given that the LA-investor’s preference nests the EZ-investor’s preference, we write a uniform preference formulation for both investors as follows. The time $t$ utility of investor $i$

\( (= EZ, LA) \) is given by

\[ U_{i,t} = H_i \left[ C_{i,t}, \mu_i \left( U_{i,t+1} | I_t \right) + b_t E_t \left[ v \left( G_{i,t+1} \right) \right] \right], \tag{2} \]

where \( b_{EZ} = 0 \) and \( b_{LA} \geq 0 \). Here, \( H_i (\cdot, \cdot) \) is the aggregator function, which combines current consumption \( C_{i,t} \) and the certainty equivalent of future utility to generate current utility \( U_{i,t} \). It takes the form

\[ H_i (C, X) = \begin{cases} 
(1 - \beta_i)C^{\rho_i} + \beta_i X^{\rho_i} \right)^{1/\rho_i}, & \text{if } 0 < \rho_i < 1, \\
C^{1-\beta_i}X^{\beta_i}, & \text{if } \rho_i = 0,
\end{cases} \tag{3} \]

where \( 0 < \beta_i < 1 \) is investor \( i \)'s time patience parameter. Parameter \( \rho_i \) determines the investor’s elasticity of intertemporal substitution: \( EIS_i = 1 / (1 - \rho_i) \).

Function \( \mu_i (U_{i,t+1} | I_t) \) is the certainty equivalent of the random future utility \( U_{i,t+1} \) conditional on time \( t \) information \( I_t \), and it has the form

\[ \mu_i (U | I_t) = \begin{cases} 
[E_t(U_{i}^\zeta_i)]^{1/\zeta_i}, & \text{if } 0 \neq \zeta_i < 1, \\
\exp[E_t(\log(U))], & \text{if } \zeta_i = 0,
\end{cases} \tag{4} \]

where \( E_t (\cdot) \equiv E (\cdot | I_t) \) is the expectation operator conditional on information \( I_t \) and where parameter \( \zeta_i \) determines the investor’s risk attitude toward aggregate future utility, as the implied parameter \( RA_i = 1 - \zeta_i \) is the investor’s relative risk aversion coefficient. We assume that the investors have correct beliefs so that we can focus on the effects of differences in loss aversion.

Up to this point, the investor’s preference is entirely standard. What is non-standard is that a new term, \( b_t E_t \left[ v \left( G_{i,t+1} \right) \right] \), is added to the second argument of \( H_i (\cdot, \cdot) \), allowing the investor to get utility directly from the performance of investing in the stock. This term captures the non-consumption utility that the agent derives directly from the specific gamble of investing in the stock rather than just indirectly via this gamble’s contribution to the next period’s wealth and the resulting consumption; the latter effect has already been captured by the certainty equivalent function, \( \mu_i \left( U_{i,t+1} | I_t \right) \). To ease exposition, we refer to
this new term as *loss aversion utility*, and its components—parameter $b_i$, argument $G_{i,t+1}$ and function $v(\cdot)$—are further specified as follows.

First, parameter $b_i$ determines the relative importance of the loss aversion utility term in the investor’s preference. For the EZ-investor, $b_{EZ} = 0$, meaning that she derives no direct utility from financial wealth fluctuations. For the LA-investor, $b_{LA} > 0$, meaning that, to a certain extent, his utility depends on the outcome of his stock investment over and above what that outcome implies for total wealth.

Second, variable $G_{i,t+1}$ defines the gamble that investor $i$ is taking by investing in the risky stock. Specifically, let $W_{i,t}$ be investor $i$’s wealth at the beginning of time $t$, and let $s_{i,t}$ be the fraction of post-consumption wealth allocated to the stock. Then this investment portfolio provides the investor with a gamble represented by

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t}), \quad (5)$$

that is, the amount invested in the stock, $s_{i,t} (W_{i,t} - C_{i,t})$, multiplied by its return in excess of the risk-free rate, $R_{t+1} - R_{f,t}$. As is standard in the literature (e.g., Barberis and Huang, 2001, 2007, 2009; Gomes, 2005; Barberis and Xiong, 2009), the risk-free rate, $R_{f,t}$, is assumed to be the “reference point” determining whether a particular outcome is treated as a gain or a loss: as long as $s_{i,t} > 0$, the stock’s return is only counted as a gain (loss) if it is larger (smaller) than the risk-free rate.

Finally, function $v(\cdot)$ determines how the investor evaluates gains and losses. We follow Barberis and Huang (2007, 2009) in assuming that $v(\cdot)$ is a piecewise-linear function:

$$v(G) = \begin{cases} 
G, & \text{if } G \geq 0, \\
\lambda G, & \text{if } G < 0,
\end{cases} \quad (6)$$

with $\lambda > 1$. This function assigns positive utility to gains and negative utility to losses. More importantly, it assigns greater negative utility to losses than positive utilities to gains of the same magnitude. This feature is known as “loss aversion” in the literature, and it is the behavioral bias that the LA-investor exhibits. Parameter $\lambda$ controls the degree of loss aversion: a one-dollar loss brings the investor $\lambda > 1$ units of negative non-consumption
utility, while a one-dollar gain brings him only one unit of positive non-consumption utility.

To summarize, the economy is characterized by the following two groups of exogenous parameters: (i) technology parameters: $\theta_H$, $\theta_L$, $\pi_H$ and $\pi_L$; and (ii) preference parameters: $b_{LA}$, $\lambda$, $\{\beta_i, \rho_i, \zeta_i\}_{i=ZE,LA}$. The technology is defined by equation (1), and the preferences are defined by equations (2)-(6).

3 Equilibrium

We consider Markov equilibria in which price-dividend ratios, the risk-free rate, and the optimal consumption and portfolio decisions are all functions of a state variable and in which the state variable evolves according to a Markov process. The Markov state variable $\omega_t$ is the LA-investor’s wealth as a fraction of aggregate wealth:

$$\omega_t = \frac{W_{LA,t}}{W_{LA,t} + W_{EZ,t}}. \quad (7)$$

Intuitively, $\omega_t$ captures the state of the economy, because it determines the strength of the pricing impact of the LA-investor’s trading behavior. The reason that we can summarize the state with a single variable is that the preferences of investors are homogeneous in wealth. A Markov equilibrium is formally defined as follows.

**Definition 1** A Markov equilibrium consists of (i) a stationary price-dividend ratio function, $f : [0, 1] \to \mathbb{R}_{++}$, (ii) a risk-free rate function, $R_f : [0, 1] \to \mathbb{R}_{++}$, (iii) a pair of consumption propensity functions, $\alpha_{LA} : [0, 1] \to [0, 1]$ and $\alpha_{EZ} : [0, 1] \to [0, 1]$, (iv) a pair of stock investment policies, $s_{LA} : [0, 1] \to \mathbb{R}$ and $s_{EZ} : [0, 1] \to \mathbb{R}$, and (v) a transition function of the state variable, $\omega : [0, 1] \times \{\theta_H, \theta_L\} \to [0, 1]$, such that

(i) the consumption policy functions and the portfolio policy functions maximize investors’ preferences given the distribution of the equilibrium return processes;

(ii) good and securities markets clear; and

(iii) the transition function of the state variable is generated by investors’ optimal decisions and the exogenous consumption growth rate process (i.e., equation [1]).

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$^{12}$Consumption propensity is the ratio of consumption over wealth, $C_{i,t}/W_{i,t}$. 
We next go through investors’ decision problems and the market clearing conditions to construct such an equilibrium.

3.1 Investors’ Decisions

Investor $i$ chooses consumption $C_{i,t}$ and the fraction of post-consumption wealth allocated to the stock $s_{i,t}$ to maximize

$$U_{i,t} = H_i [C_{i,t}, \mu_i (U_{i,t+1} | I_t) + b_i E_t [v (G_{i,t+1})]]$$

subject to the definition of capital gains/losses in stock investment

$$G_{i,t+1} = s_{i,t} (W_{i,t} - C_{i,t}) (R_{t+1} - R_{f,t})$$

and to the standard budget constraint

$$W_{i,t+1} = (W_{i,t} - C_{i,t}) M_{i,t+1},$$

where

$$M_{i,t+1} = R_{f,t} + s_{i,t} (R_{t+1} - R_{f,t})$$

is the gross return on the investor’s portfolio, and functions $H_i (\cdot, \cdot)$, $\mu_i (\cdot)$, and $v (\cdot)$ are given by equations (3), (4), and (6), respectively.

For brevity, we only derive the first-order conditions characterizing the investor’s optimal decisions for the case of a non-unit EIS (i.e., for the case of $\rho_i \neq 0$ in the aggregator function $H_i (\cdot, \cdot)$). The first-order conditions for the case of a unit EIS are relegated to Appendix A.

The Bellman equation of the investor’s problem is

$$U_{i,t} \equiv J_i (W_{i,t}, I_t)$$

$$= \max_{C_{i,t}, s_{i,t}} [(1 - \beta_i) C_{i,t}^{\rho_i} + \beta_i [\mu_i (J (W_{i,t+1}, I_{t+1}) | I_t) + b_i E_t [v (G_{i,t+1})])^{\rho_i}]^{1/\rho_i}.$$ 

Because functions $H_i (\cdot, \cdot)$, $\mu_i (\cdot)$, and $v (\cdot)$ are all homogeneous of degree one, the indirect
value function $J_i(W_{i,t}, I_t)$ is also homogeneous of degree one:

$$J_i(W_{i,t}, I_t) = A_i(I_t) W_{i,t} \equiv A_{i,t} W_{i,t}.$$ 

Therefore,

$$A_{i,t} W_{i,t} = \max_{C_{i,t}, s_{i,t}} \left[ (1 - \beta_i) C_{i,t}^\rho_i + \beta_i (W_{i,t} - C_{i,t})^\rho_i \right] \mu_i (A_{i,t+1} M_{i,t+1} | I_t) + b_i E_t [v (s_{i,t} (R_{t+1} - R_{f,t}))]^{\rho_{\pi}} \right]^{1/\rho_i},$$

which implies that the consumption and portfolio decisions are separable.

In particular, the portfolio decision is determined by

$$B_{i,t}^* = \max_{s_{i,t}} \left[ \mu_i (A_{i,t+1} M_{i,t+1} | I_t) + b_i E_t [v (s_{i,t} (R_{t+1} - R_{f,t}))] \right],$$

and after defining the consumption propensity as

$$\alpha_{i,t} = C_{i,t}/W_{i,t},$$

the consumption decision is made based on

$$A_{i,t} = \max_{\alpha_{i,t}} \left[ (1 - \beta_i) \alpha_{i,t}^\rho_i + \beta_i (1 - \alpha_{i,t})^\rho_i \left( B_{i,t}^*\right)^{\rho_i} \right]^{1/\rho_i}. \quad (10)$$

The first-order condition for optimal consumption propensity $\alpha_{i,t}^*$ is $^{13}$

$$B_{i,t}^* = \left( \frac{1 - \beta_i}{\beta_i} \right)^{1/\rho_i} \left( \frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1-1/\rho_i}.$$ \quad (11)

Combining equations (10) and (11) delivers

$$A_{i,t} = (1 - \beta_i)^{1/\rho_i} \left( \alpha_{i,t}^* \right)^{1-1/\rho_i},$$

$^{13}$ All of the first-order conditions of the investor’s problem are both necessary and sufficient, as the objective functions are concave.
which, by the recursive structure, in turn implies
\[ A_{i,t+1} = (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t+1}^*)^{1-1/\rho_i}. \] (12)

Substituting equations (11) and (12) into equation (9) gives the following single program, which summarizes the investor’s consumption and portfolio decisions:
\[
\left( \frac{1 - \beta_i}{\beta_i} \right)^{1/\rho_i} \left( \frac{\alpha_{i,t}^*}{1 - \alpha_{i,t}^*} \right)^{1-1/\rho_i} = \max_{s_i,t} \left\{ \mu_i \left[ (1 - \beta_i)^{1/\rho_i} (\alpha_{i,t+1}^*)^{1-1/\rho_i} M_{i,t+1}[I_t] \right] + b_i E_t [v(s_{i,t} (R_{t+1} - R_{f,t}))] \right\}. \] (13)

As a consequence, solving the investor’s partial-equilibrium problem boils down to solving a fixed-point problem defined by the first-order condition and the value function of the above maximization problem. In the Markov equilibrium, the investor’s consumption policy and investment policy are both functions of the state variable \( \omega_t, s_i (\cdot) \) and \( \alpha_i (\cdot) \); the first-order condition and the value function of program (13) thus form a system of two equations with these two unknown functions; given the equilibrium asset return processes \( (R_{t+1} \text{ and } R_{f,t}) \), these partial equilibrium optimal policies can be computed from this system.

It needs certain carefulness to derive the first-order conditions for the portfolio choice, as the utility function, \( v(\cdot) \), the function that the investor uses to evaluate gains/losses, is not differentiable everywhere but instead has a kink at the origin. As will become clear in the subsequent analysis, it is this non-differentiability at the origin that is responsible for the non-participation of the LA-investor in the stock market. Formally, the optimal stock investment \( s_{i,t}^* \) is characterized by the following conditions:\(^{14}\)
\[
FOC_{i,+} \equiv (1 - \beta_i)^{1/\rho_i} \left[ E_t \left( \alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i} \right) \right]^{1/\zeta_i - 1} E_t \left[ \alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t}) \right] + b_i E_t [v(R_{t+1} - R_{f,t})] = 0, \text{ for } s_{i,t}^* > 0, \]
\[
(14)
\]

\(^{14}\)To be precise, these conditions apply to the case of a non-unit risk aversion, i.e., they are true when \( RA_i \neq 1 \text{ or } \zeta_i \neq 0 \) in the certainty-equivalent function \( \mu(\cdot) \). As for the case of a unit risk aversion, simply replace the first terms with \( (1 - \beta_i)^{1/\rho_i} e^{(1-1/\rho_i)E_t \log(\alpha_{i,t+1}^*)} \).
\[
FOC_{i,-} \equiv (1 - \beta_i)^{1/\rho_t} \left[ E_t \left( \alpha_{i,t+1}^{(1-1/\rho_t)} \kappa_i M_{i,t+1}^{\zeta_i} \right) \right]^{1/\zeta_i-1} E_t \left[ \alpha_{i,t+1}^{(1-1/\rho_t)} \kappa_i M_{i,t+1}^{\zeta_i-1} (R_{t+1} - R_{f,t}) \right] \\
- b_i E_t [v (R_{f,t} - R_{t+1})] = 0, \text{ for } s_{i,t}^* < 0, \quad (15)
\]

\[
FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s_{i,t}^* = 0. \quad (16)
\]

In particular, as for the EZ-investor, the expressions of \(FOC_{i,+}\) and \(FOC_{i,-}\) are the same because \(b_{EZ} = 0\). Therefore, her first-order conditions are reduced to the following equation:

\[
E_t \left[ \alpha_{EZ,t+1}^{(1-1/\rho_{EZ})} \kappa_{EZ} M_{EZ,t+1}^{\zeta_{EZ}-1} (R_{t+1} - R_{f,t}) \right] = 0. \quad (17)
\]

### 3.2 Stock Prices and Wealth Dynamics

In this subsection, we rely on market-clearing conditions to derive the expression of price-dividend ratios \(f_t \equiv P_t / D_t\) and the evolution of the state variable \(\omega_t\).

The good market-clearing condition is

\[
C_{EZ,t} + C_{LA,t} = D_t. \quad (18)
\]

Using the definition of consumption propensity, we can express the consumption levels as products of consumption propensity functions and individual wealth levels:

\[
C_{EZ,t} = \alpha_{EZ} (\omega_t) W_{EZ,t} \text{ and } C_{LA,t} = \alpha_{LA} (\omega_t) W_{LA,t}.
\]

Then, substituting the above expressions into the good-market clearing condition gives

\[
\alpha_{EZ} (\omega_t) W_{EZ,t} + \alpha_{LA} (\omega_t) W_{LA,t} = D_t. \quad (19)
\]

Let \(W_t = W_{EZ,t} + W_{LA,t}\) be the aggregate wealth of the whole economy at time \(t\). Recall that the definition of \(\omega_t\) in equation (7) implies that \(W_{EZ,t} = (1 - \omega_t) W_t\) and \(W_{LA,t} = \omega_t W_t\). Therefore, equation (19) becomes

\[
[\alpha_{EZ} (\omega_t) (1 - \omega_t) + \alpha_{LA} (\omega_t) \omega_t] W_t = D_t,
\]

15
which implies
\[ W_t = \frac{D_t}{\alpha_{EZ}(\omega_t) (1 - \omega_t) + \alpha_{LA}(\omega_t) \omega_t}. \] 
(20)

Because the bond is zero net supply, and the stock has a net supply of one share, the aggregate economy wealth is also equal to the stock price plus its dividend:
\[ W_t = P_t + D_t. \] 
(21)

Combining equations (20) and (21) gives the price-dividend ratio function:
\[ f(\omega_t) = \frac{(1 - \omega_t) \alpha_{EZ}(\omega_t)}{(1 - \omega_t) \alpha_{EZ}(\omega_t) + \omega_t \alpha_{LA}(\omega_t)} \frac{1 - \alpha_{EZ}(\omega_t)}{\alpha_{LA}(\omega_t) \omega_t} + \frac{\omega_t \alpha_{LA}(\omega_t)}{(1 - \omega_t) \alpha_{EZ}(\omega_t) + \omega_t \alpha_{LA}(\omega_t)} \frac{1 - \alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}. \] 
(22)

Equation (22) says that the price-dividend ratios in the heterogeneous agent economy are equal to a weighted average of two terms: \( \frac{1 - \alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)} \) and \( \frac{1 - \alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)} \). The expressions of these two terms correspond to the price-dividend ratios in the representative agent economies populated only by the EZ-investor and by the LA-investor, respectively.\(^{15}\) So, roughly speaking, the price-dividend ratios in a heterogeneous economy is the weighted average of the price-dividend ratios in representative agent economies, although the weight is not simply the wealth share but is instead a rather complicated expression related to the wealth share and investors’ optimal consumption policies.

Given the price-dividend ratio function \( f_t = f(\omega_t) \) and the Markov structure of the state variable evolution \( \omega_{t+1} = \omega(\omega_t, \theta_{t+1}) \), the distribution of stock returns \( R_{t+1} \) also has a Markov structure and is determined by
\[ R(\omega_t, \theta_{t+1}) \equiv R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t/D_t} + 1 \frac{D_{t+1}}{D_t} = f(\omega(\omega_t, \theta_{t+1})) + 1 \frac{\theta_{t+1}}{f(\omega_t)}. \] 
(23)

We now turn to examine how the state variable, \( \omega_t \), evolves over time. The gross return

\(^{15}\)To see this, note that, in a representative agent economy, the agent holds the whole share of the stock and consumes the entire dividend, which means that \( \alpha_{i,t} W_{i,t} = \alpha_{i,t} (P_t + D_t) = D_t \) and thus \( P_t/D_t = (1 - \alpha_{i,t})/\alpha_{i,t} \).
to the LA-investor’s optimal portfolio is

\[ M_{LA}(\omega_t, \theta_{t+1}) \equiv M_{LA,t+1} = R_{f,t} + s_{LA,t} (R_{t+1} - R_{f,t}) \]

\[ = R_f(\omega_t) + s_{LA}(\omega_t) [R(\omega_t, \theta_{t+1}) - R_f(\omega_t)]. \quad (24) \]

Therefore, the LA-investor’s next period wealth is

\[
W_{LA,t+1} = \frac{1 - \alpha_{LA}(\omega_t)\omega_t M_{LA}(\omega_t, \theta_{t+1})}{\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t} D_t,
\]

where the second equation follows from \( W_{LA,t} = \omega_t W_t \) and equation (20).

Applying equation (20) one period forward gives

\[
W_{t+1} = \frac{D_{t+1}}{\alpha_{EZ}(\omega_{t+1})(1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1})\omega_{t+1}}. \quad (25)
\]

Combining equations (25) and (26) and recalling the definition of \( \omega_{t+1} = \frac{W_{LA,t+1}}{W_{t+1}} \) and \( \theta_{t+1} = \frac{D_{t+1}}{D_t} \), we have

\[
\omega_{t+1} = \frac{[1 - \alpha_{LA}(\omega_t)]\omega_t M_{LA}(\omega_t, \theta_{t+1})[\alpha_{EZ}(\omega_{t+1})(1 - \omega_{t+1}) + \alpha_{LA}(\omega_{t+1})\omega_{t+1}]}{[\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t] \theta_{t+1}}, \quad (27)
\]

which implicitly determines the evolution of \( \omega_t: \omega_{t+1} = \omega(\omega_t, \theta_{t+1}) \).

Finally, substituting \( W_{EZ,t} = (1 - \omega_t) W_t, W_{LA,t} = \omega_t W_t \) and equation (20) into the stock-market clearing condition,

\[ P_t = s_{EZ,t}^*(1 - \alpha_{EZ,t}^*) W_{EZ,t} + s_{LA,t}^*(1 - \alpha_{LA,t}^*) W_{LA,t}, \]

we link investors’ policy functions to the price-dividend ratio function as follows:

\[
f(\omega_t) = \frac{s_{EZ}(\omega_t) [1 - \alpha_{EZ}(\omega_t)](1 - \omega_t) + s_{LA}(\omega_t) [1 - \alpha_{LA}(\omega_t)]\omega_t}{\alpha_{EZ}(\omega_t)(1 - \omega_t) + \alpha_{LA}(\omega_t)\omega_t}. \quad (28)
\]

To summarize, computing the equilibrium is involved with solving the seven unknown functions, \( f(\cdot), R_f(\cdot), \alpha_{LA}(\cdot), \alpha_{EZ}(\cdot), s_{LA}(\cdot), s_{EZ}(\cdot), \) and \( \omega(\cdot, \cdot) \) from the system formed.
by equations (13)-(16), (22)-(24), (27) and (28). This system consists of seven independent equations: two value functions (equation [13] for \( i = EZ, LA \)), two first-order conditions (one of equations [14]-[16] for \( i = EZ, LA \)), two market clearing conditions (equations [22] and [28]), and a state variable evolution function (equation [27]). Equations (23) and (24) are intermediate steps for calculating the wealth dynamics.

Two remarks are in order. First, although the market is complete in the present project, the standard Pareto efficiency technique commonly used in the market-selection literature (e.g., Blume and Easley, 2006; Yan, 2008; Kogan, Ross, Wang and Westerfield, 2011; Borovička, 2012) cannot be applied here, because the LA-investor's preference is non-differentiable and depends not only on the intertemporal consumption plans but also on the endogenous stock return process \textit{per se}, thereby making it necessary to explicitly solve the equilibrium. We therefore develop an algorithm based on Kubler and Schmedders (2003) to compute the Markov equilibrium and use simulations to analyze the survival and price impact of the LA-investor. The details of the algorithm are delegated to Appendix B.

Second, our analysis ignores the issue of the existence and uniqueness of the equilibrium. As is well-known in the literature, it is hard to establish the general results on the existence and uniqueness of the equilibria in heterogeneous agent models. Therefore, in the present paper, we simply start the analysis under the assumption that an equilibrium exists and use numerical methods to find this equilibrium. Rigorously speaking, a numerical method can never find the exact equilibrium; what it finds, if any, is the “\( \varepsilon \)-equilibrium” defined by Kubler and Schmedders (2003), who interpret the computed \( \varepsilon \)-equilibrium as an approximate equilibrium of some other economy with endowments and preferences that are close to those in the original economy.

4 Loss Aversion and Market Selection

In this section, we first in Subsection 4.1 analyze the representative agent economies, that is, economies populated by homogeneous investors. This analysis serves two purposes. First, it verifies the result that loss aversion raises equity premiums, which is well-known in the literature (e.g., Benartzi and Thaler, 1995; Barberis, Huang and Santos, 2001). Second, it provides a useful springboard for our analysis of the heterogeneous agent economies, as it
helps to develop the intuitions for how loss aversion changes an investor’s investment and saving behaviors.

We then move to the more realistic economies populated by both the EZ-investor and the LA-investor and apply the algorithm in Appendix B to numerically compute the equilibrium price functions \( (f(\cdot) \text{ and } R_f(\cdot)) \), policy functions \( (\alpha_{LA}(\cdot), \alpha_{EZ}(\cdot), s_{LA}(\cdot) \text{ and } s_{EZ}(\cdot)) \), and the state variable transition function \( (\omega(\cdot, \cdot)) \). We use simulations to show how loss aversion affects the investor’s wealth accumulation and pricing impact both in the short run and in the long run via portfolio decisions in Subsection 4.2 and via saving behaviors in Subsection 4.3. To isolate the role of loss aversion, in these two subsections, we assume that both investors have otherwise identical preferences except that the LA-investor derives loss aversion utility and the EZ-investor does not.

Before solving the models, we need to calibrate the parameter values. Because we are interested in the implications of preferences, we allow the preference parameters to vary over a certain range while fixing the four technology parameters in equation (1) for all computations and simulations. We interpret one period as one month and follow Mehra and Prescott (1985) in setting \( \pi_H = \pi_L = \frac{1}{2} \) so that the economy is in booms and recessions with equal probability.\(^{16}\) Based on the data spanning the 20th century, the historical mean and volatility of the log annual consumption growth process are \(1.84\% \text{ and } 3.79\%\), respectively (see Barberis and Huang (2009)). To match these two moments, we set \( \theta_H = 1.0126 \) and \( \theta_L = 0.9906 \). Table 1 summarizes our choice of technology parameters.

### 4.1 Representative Agent Economy

In this subsection, we assume that the EZ-investor and the LA-investor have identical preferences; that is, \( \beta_{EZ} = \beta_{LA} \equiv \beta, \rho_{EZ} = \rho_{LA} \equiv \rho, \zeta_{EZ} = \zeta_{LA} \equiv \zeta \text{ and } b_{EZ} = b_{LA} \equiv b. \) As a result, the economy is the well-studied representative agent economy.

\(^{16}\)Some studies have assumed that loss-averse investors evaluate investment performance on an annual frequency (e.g., Benartzi and Thaler 1995; Barberis, Huang and Santos, 2001). Our results are valid if we take one period as one year in our economy.
In this case, the representative agent has to hold the stock in equilibrium, so that the first-order condition given by equation (14) with \( M_{t,t+1} = R_{t+1} \) defines the optimality of the investor’s investment decision. As mentioned in the discussions in footnote 15, the consumption good market-clearing condition links the price-dividend ratios \( f_t \) to the optimal consumption policy \( \alpha_t \) as follows:

\[
D_t = (1 - \alpha_t) (P_t + D_t) \Rightarrow f_t = \frac{1 - \alpha_t}{\alpha_t}.
\] (29)

Therefore, equations (13), (14) and (29) define a system for three unknowns: \( f_t, \alpha_t \) and \( R_{f,t} \). Given the i.i.d. investment opportunities, we conjecture that

\[
(f_t, \alpha_t, R_{f,t}) = (f, \alpha, R_f), \quad \forall t.
\] (30)

The problem can be easily solved using any non-linear solver.

Table 2 reports the annualized continuously compounded equilibrium equity premiums \( (EP^u = 12 [E (\log R_{t+1}) - \log R_f]) \), risk-free rates \( (r_f^u = 12 \log (R_f)) \) and consumption policies \( (\alpha^u = 12 \alpha) \) for a variety of combinations of preference parameter values. For all combinations, we hold constant the time patience parameter \( \beta \), the loss-aversion parameter \( \lambda \) and the relative risk-aversion coefficient \( RA: \beta = 0.998, \lambda = 2.25 \) and \( RA = 1 \) (or \( \zeta = 0 \)). The choice of \( \beta \) is borrowed from Bansal and Yaron (2004) and it corresponds to an annual time-patience parameter of \( 0.976 = \beta^{12} \). The choice of \( \lambda \) is based on the estimation of Tversky and Kahneman (1992). When EIS is equal to one (i.e., \( \rho = 0 \)) and when there is no loss aversion utility (i.e., \( b = 0 \)), setting \( RA = 1 \) (or \( \zeta = 0 \)) reduces the investor’s preference to an expected log utility, which is an important benchmark case that the market-selection literature has been focusing on (Breiman, 1961; Hakansson, 1971; De Long, Shleifer, Summers and Waldmann, 1991; Blume and Easley, 1992).

[INSERT TABLE 2 HERE]

Panels A, B and C correspond to different values of EIS: \( EIS = 1 (\rho = 0), EIS = 0.8 (\rho = -0.25) \) and \( EIS = 1.2 (\rho = 1/6) \). To check the role of loss aversion, in each
panel, parameter \( b \), which controls the relative importance of the loss aversion utility in the investor’s preference, is set at three different values: \( b = 0, b = 0.0005 \) and \( b = 0.001 \). When \( b = 0 \), the investor’s preference does not exhibit loss aversion, and this economy has been well studied in the literature (e.g., Weil, 1989). When \( b > 0 \), the investor’s preference exhibits loss aversion; such an economy is the focus of behavioral finance, such as Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), and Barberis and Huang (2007, 2009). The choice of both positive values of \( b \) in the table, 0.0005 and 0.001, is justified by the investor’s attitudes to independent large and small monetary gambles: both parameterizations of the investor’s preference satisfy Barberis and Huang’s conditions L and S (2007, p 217 and p 219).\(^{17}\) The last two columns of Table 2 report the premiums the representative agent would pay to avoid a large gamble and a small gamble, which are computed according to equation (34) in Barberis and Huang (2009, p 1566).

Three notable patterns show up in Table 2. The first pattern regards the equity premium. In all three panels, when \( b = 0 \), that is, when loss aversion is absent in the investor’s preference, the equity premium is quite small (0.07\%) relative to its historical value (6\%), which is the well-known equity premium puzzle. Once loss aversion is introduced, the equity premiums are raised significantly. Say, when \( b = 0.001 \), the model can generate an equity premium as high as 3.01\%, which is more than 40 times the equity premium corresponding to an economy populated by only EZ-investors. The increased equity premiums still fall short of the empirical value, as in our model, the stock is a claim to the smooth aggregate consumption process, and, as a result of the constant equilibrium price-dividend ratios in equation (30), the stock returns are not volatile enough to cause the loss-averse investor to be scared of holding the stock.\(^{18}\) As mentioned before in Section 2, this mismatch between the model-generated equity premium and the historical equity premium does not have any

\(^{17}\)The literature cares about investors’ attitudes to independent monetary gambles, as it was, in part, the difficulty that researchers encountered in reconciling the equity premium with these attitudes that launched the equity premium literature in the first place. Barberis and Huang’s (2007) condition L is: “An individual with wealth of $75,000 should not pay a premium higher than $15,000 to avoid a 50:50 chance of losing $25,000 or gaining the same amount.” Their condition S is: “An individual with wealth of $75,000 should not pay a premium higher than $40 to avoid a 50:50 chance of losing $250 or gaining the same amount.”

\(^{18}\)Barberis, Huang and Santos (2001) also study the pricing impact of loss aversion in a representative agent economy with dividends equal to consumption, and they report an equity premium of 1.26\% as the relative risk aversion coefficient is equal to 1 (see the top part of their Table II), which is close to the equity premium generated in our model (1.22\% when \( b = 0.0005 \)).
impact on our analysis. What really matters is that the LA-investor exhibits first-order risk aversion and is more reluctant to hold the stock than the EZ-investor, which is exactly the reason why behavioral finance introduces loss aversion to explain the equity premium puzzle.

The second pattern concerns the risk-free rate. In all three panels, the risk-free rate decreases with $\beta$. This occurs because as the investor is more concerned about fluctuations in the value of his financial wealth and as he is more loss-averse, he is more inclined to allocate wealth to the safe asset to avoid the potential painful losses associated with the risky asset. This suggests that in a heterogeneous agent economy populated by both the LA-investor and the EZ-investor, the bond is more attractive to the former than to the latter.

The third pattern is about the consumption policy. When EIS is equal to one, the investor’s monthly saving ratio is optimally chosen to be equal to the time patience parameter, $\beta$. Therefore, in Panel A, the optimal consumption propensity $\alpha$ is independent of parameter $b$. However, when EIS is different from 1, $\alpha$ varies with $b$: $\alpha$ decreases (increases) with $b$ when EIS is less (greater) than 1 in Panel B (Panel C). As is standard in the portfolio choice problem for recursive preferences, two forces—the income effect and the substitution effect—are at play here. The asymmetric treatment of losses from gains in the loss aversion utility tends to lower the value, measured in utility terms, of the investor’s future investment opportunities; that is, a higher $b$ tends to yield a lower $B_i^*$ in equation (9). This lowered $B_i^*$ has two effects on current consumption: it lowers consumption propensity through the income effect but raises consumption propensity through the substitution effect. When EIS is below 1, the income effect dominates, so that $\alpha$ decreases with $b$; when EIS is above 1, the substitution effect dominates, and the dependence of $\alpha$ on $b$ reverses as a result. The different responses of $\alpha$ to $b$ in different cases of EIS suggest that how loss aversion affects the LA-investor’s survival might depend on whether EIS is greater than or smaller than 1, as the literature suggests that saving behavior is a key determinant on survival. This issue will be examined in Subsection 4.3.

4.2 EIS=1: Portfolio Selection

In this subsection, we study the heterogeneous agent economy and fix EIS at 1, so that both investors optimally choose to have a constant monthly consumption-wealth ratio: $\alpha_{i,t}^*$ =
1 − β_i, for i = EZ, LA. To isolate the impact of loss aversion, we still assume that the preferences of both investors are otherwise identical except that the LA-investor derives loss aversion utility, while the EZ-investor does not. So, except that β_{LA} > 0, β_{EZ} = 0, all other parameters are the same across investors: β_{EZ} = β_{LA} \equiv β, ρ_{EZ} = ρ_{LA} \equiv ρ and ζ_{EZ} = ζ_{LA} \equiv ζ. The assumption of a common time-patience parameter implies that both investors have the same endogenous saving rate, which is equal to β. The focus of this subsection is therefore essentially how loss aversion changes the LA-investor’s portfolio decision, which in turn affects the LA-investor’s wealth accumulation and pricing impacts in a complete financial market.

4.2.1 Wealth Dynamics and Survival

We follow the market selection literature, such as Yan (2008) and Kogan, Ross, Wang and Westerfield (2009), in deﬁning the “extinction”, “survival” and “dominance” of the LA-investor in terms of his wealth shares as follows.

**Definition 2** The LA-investor is said to become extinct or vanish if

\[ \lim_{t \to \infty} \omega_t = 0, \text{ almost surely (a.s.);} \]

to survive if extinction does not occur; and to dominate the market if

\[ \lim_{t \to \infty} \omega_t = 1, \text{ a.s..} \]

Our subsequent analysis suggests that the LA-investor will vanish via the channel of portfolio decisions if loss aversion causes him to be further from the log investor in terms of risk attitude than the EZ-investor. We also show that empirically relevant parameter values typically lead to this result and that the selection process is excessively slow.

To illustrate how the LA-investor’s wealth shares (ω_t) evolve over time, Table 3 reports their medians at times t = 60, 120, and 600 months (i.e., 5, 10 and 50 years) when the LA-investor has initial wealth shares of ω_0 = 0.5 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). We report the results for various values

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19 The evolution patterns of ω_t are robust to the choice of ω_0.
of \( \lambda \), which controls the degree of loss aversion. The technology parameters are fixed at the values in Table 1. The time patience parameters are \( \beta_{EZ} = \beta_{LA} = 0.998 \) and parameter \( b_{LA} \) is set as 0.001. The medians of \( \omega_t \) are obtained from simulations. We first use the algorithm described in Appendix B to solve the equilibrium state transition function \( \omega(\cdot, \cdot) \) and then use it to simulate \( N = 5000 \) economies. For each economy, we make \( T = 600 \) independent draws of \( \theta_{t+1} \) from the distribution described in equation (1) to simulate a time series \( \{\theta_{t+1}\}_{t=1}^{T} \). We then use the solved function \( \omega(\cdot, \cdot) \) to calculate the next-period state \( \omega_{t+1} \). Finally the medians of \( \omega_t \) are estimated from the 5,000 simulated sample paths at time \( t \).

We first note that the speed of wealth changes is excessively slow. In general, after 50 years (at \( t = 600 \)), on a typical sample path, \( \omega_t \) still maintains above 70% of its initial value. This suggests that the selection mechanism is not effective in terms of wealth, which is consistent with Yan (2008) who shows that it takes hundreds of years for an investor with incorrect beliefs to lose half of his wealth share in an economy populated with heterogeneous CRRA investors.

In terms of long-run survival, the results in Table 3 suggest that the insight in De Long, Shleifer, Summers and Waldman (1991) and Blume and Easley (1992) holds for recursive preferences in a general equilibrium setting: whether an investor survives depends on how close his/her preference is to log utility whose objective is to maximize expected wealth growth. In Panel A of Table 3, the EZ-investor is the log investor, since \( RA_{EZ} = EIS_{EZ} = 1 \).

We can see that for all values of \( \lambda \), the LA-investor’s wealth shares shrink as time passes. This suggests that in an economy populated with the log investor and the LA-investor, the LA-investor always loses out.

In Panel B of Table 3, we change the relative risk aversion coefficient of both investors from 1 to 3, so that the EZ-investor is no longer the log investor. In this case, we observe that the LA-investor sometimes survives, while at other times, he vanishes: when \( \lambda = 1.05 \), the LA-investor’s wealth shares increase over time, while when \( \lambda = 1.5, 2.25, \) or 3, his wealth shares decrease over time. When \( RA_{EZ} = 3 \), the EZ-investor is more risk averse than the log investor. If \( \lambda \) is close to 1, the loss aversion utility in the LA-investor’s preference is close to
the expected gains/losses (i.e., \(b_{LA}E_t[v(G_{L,A,t+1})] \approx b_{LA}E_t(G_{L,A,t+1})\)). This causes the LA-investor’s preference as if to be generated from combining the EZ-investor’s preference and the risk neutral investor’s preference, leading the LA-investor to hold portfolios corresponding to a greater risk tolerance. Therefore, the LA-investor can potentially be closer to the log investor in terms of risk attitude than the EZ-investor, which explains his long-run survival for the case of \(\lambda = 1.05\). On the other hand, if \(\lambda\) is much greater than 1, as we believe is empirically likely, loss aversion penalizes losses much stronger than it rewards gains, making the LA-investor reluctant to invest in the volatile stocks. This causes the LA-investor to mimic an investor who is more risk averse, and hence further from log utility, than the EZ-investor. Therefore, the LA-investor vanishes in the long run.

Table 4 further verifies the above intuitions. It is difficult to directly compare the risk attitude of the LA-investor to that of the EZ-investor (and of the log investor). This is because, unlike the EZ-investor whose risk attitude is solely determined by parameter \(\zeta_{EZ}\), the LA-investor’s risk attitude is jointly determined, in a very complicated way, by parameter \(\zeta_{LA}\), which captures traditional smooth risk aversion, and by parameters \(b_{LA}\) and \(\lambda\), which capture first-order risk aversion due to loss aversion. Nonetheless, we can compare their risk attitudes indirectly by examining risk premiums generated by representative agent economies—the equilibrium equity premium should be high if the representative agent avoids risk very much. Table 4 adopts such an approach.

[INSERT TABLE 4 HERE]

Specifically, the first row of Table 4 reports the value of the equity premium, \(EP_{LA}\), in the representative agent economy populated with the LA-investor for various values of \(\lambda\), while other model parameters take the same values as those in Panel B of Table 3. The second row compares \(EP_{LA}\) to \(EP_{log} (=1.20 \text{ basis points})\), which is the equity premium emerging from the representative log investor economy. We find that \(EP_{LA}\) is greater than \(EP_{log}\) for all values of \(\lambda\), suggesting that the LA-investor behaves as if he is more risk averse than the log investor. The third row compares \(EP_{LA}\) to \(EP_{EZ} (=3.60 \text{ basis points})\), the equity premiums emerging from the representative agent economy populated with the EZ-investor. For \(\lambda = 1.05\), we find that \(EP_{LA} < EP_{EZ}\), suggesting the LA-investor behaves as if he is less
risk averse than the EZ-investor. Given that the EZ-investor is more risk averse than the log investor \((RA_{EZ} = 3 > 1)\), we now have the complete order among the three investors—i.e., for the case of \(\lambda = 1.5\), the EZ-investor is more risk averse than the LA-investor, who is in turn more risk averse than the log investor. As a result, the LA-investor is closer to the log investor and survives in the long run. For other cases, we can conduct a similar analysis and found that the ordering between the LA-investor and EZ-investor reverses, which implies that the LA-investor’s preference is further from the log utility and hence he eventually vanishes.

So far, our analysis on wealth shares suggests that the implication of loss aversion for wealth dynamics (and hence the long-run survival) can be largely understood in a way of interpreting the LA-investor as a traditional investor who has a high risk aversion coefficient. However, loss aversion and traditional smooth risk aversion are not observationally equivalent, because they have dramatically different implications for price impact dynamics, which we will analyze next.

### 4.2.2 Price Impacts

The effectiveness of the market selection mechanism should be judged by how the price impacts and not the wealth shares of the LA-investor change over time, as what one really cares about is whether the behavior of asset prices can largely be captured by models without the LA-investor. We use the ratio of the conditional equity premium \(EP_t\) in the heterogeneous agent economy to that in the representative agent economy with only the EZ-investor, \(EP_{EZ}\), to capture the price impact of the LA-investor at state \(\omega_t\); that is,

\[
priceimpact (\omega) = \frac{E(R_{t+1} - R_{f,t} | \omega = \omega)}{E(R_{t+1} - R_{f,t} | \omega = 0)}.
\]  

Although we focus on the implication of loss aversion for equity premiums, we believe that our intuitions can be extended to its implications for other dimensions of asset prices. For example, Barberis, Huang and Santos (2001) rely on dynamic loss aversion (i.e., a combination of loss aversion and “the house money effect”) to generate excess volatility of stock returns in a representative agent economy. If loss aversion does not affect equity premiums in the heterogeneous agent economy in the long run, there is no reason to believe that
introducing “the house money effect” will produce excess volatility in such an economy.

Table 5 reports the medians of \( \text{priceimpact}_t \equiv E P_t / E P_{EZ} \) in a variety of economies. The parameters take the same values as those in Table 3. Unlike Table 3, Table 5 only reports results for the case of \( \lambda = 2.25 \) and not for the cases of \( \lambda = 1.05, 1.5 \) and 3: the case of \( \lambda = 1.05 \) is not empirically plausible, while the results in cases of \( \lambda = 1.5 \) and 3 are similar to those in the case of \( \lambda = 2.25 \).\(^{20}\) In addition, unlike Table 3 where the LA-investor starts with half of the aggregate wealth, Table 5 assumes that the LA-investor’s initial wealth shares can take four possible values, \( \omega_0 = 0.1, 0.5, 0.9 \) or 1, because, as will become clear shortly, we will use these different \( \omega_0 \)’s to illustrate different aspects of the effectiveness of the market selection mechanism. In particular, when \( \omega_0 = 1 \), the economy becomes a representative agent economy with only the LA-investor, which generates the largest price impact of loss aversion.

[INSERT TABLE 5 HERE]

We observe that in all cases, as time passes, the LA-investor gradually loses his impact on the equity premiums, because the medians of \( \text{priceimpact}_t \) gradually decline. So, the LA-investor is unlikely to have price impacts in the long run. This result is not surprising given that, according to Table 3, the LA-investor’s wealth shares shrink over time for parameterizations considered in Table 5.

In terms of the selection speed of price impacts, the market selection mechanism is quite effective for the following two reasons. First, introducing the EZ-investor can immediately reduce the price impact of loss aversion. For example, in Panel A of Table 5, the first entry of the last column shows that \( \text{priceimpact}_0 \) is around 19 when \( \omega_0 = 1 \); that is, in representative agent economies with the LA-investor, loss aversion raises equity premiums by as much as 19 times the value produced by Epstein-Zin preferences. However, in the heterogeneous agent economy, when \( \omega_0 = 0.5 \), \( \text{priceimpact}_0 \) drops sharply to a value around 2 at time 0.\(^{21}\) Second, even when \( \omega_0 \) is high and the price impact of loss aversion is initially big, it is likely that a small drop in wealth shares leads to a large drop in price impact. For instance, in

\(^{20}\)In Table 3, we are interested in these three cases of \( \lambda = 1.05, 1.5 \) and 3, because we want to understand whether the insight on survival in Blume and Easley (1992) can be extended to the recursive preferences.

\(^{21}\)This is consistent with Chapman and Polkovnichenko (2009) who show that the risk premium is sensitive to ignoring heterogeneity in a static model in which investors exhibit first-order risk aversion.
Panel A of Table 5, at time 0 when $\omega_0 = 0.9$, $priceimpact_0$ is around 10. However, after five years ($t = 60$), it drops dramatically to a value around 7 and after 50 years ($t = 600$), it further drops to a value around 3.

The reason underlying the above strong selection force in terms of price impacts is as follows. In the heterogeneous agent economy, the LA-investor might optimally choose not to participate in the stock market, so that the conditional equity premium is determined by the first-order condition of the EZ-investor and not directly affected by loss aversion, leading to a substantially lower equity premium. As a further illustration, we plot in Figure 1 the price impact function $priceimpact(\cdot)$ and the LA-investor’s investment policy function $s_{LA}(\cdot)$ for the parameter configuration in Panel A of Table 5. There is a kink in the price impact function in Panel (a), and the location of the kink of $priceimpact_t$ is determined by the level of the wealth share at which the LA-investor starts to buy the stock in Panel (b). The price impact function is rather steep around the kink, and therefore, declining $\omega_t$ from 1 toward 0 would lead to a large drop of $priceimpact_t$. This explains the effectiveness of market selection in terms of price impacts.

[INSERT FIGURE 1 HERE]

So, in our economy, wealth and price impact do not move in the same order—the selection process is excessively slow in terms of wealth, but it is very fast in terms of price impacts. This differentiates our model from other selection models using traditional CRRA preferences which show that the selection speed is the same no matter whether it is measured in terms of wealth or price impacts (e.g., Yan 2008; Fedyk, Heyerdahl-Larsen and Walden, 2012). The difference between wealth dynamics and price impact dynamics highlighted by our analysis also complements Kogan, Ross, Wang and Westerfield (2006, 2011) and Cvitanić and Malamud (2011) who underscore that survival and price impact are two distinct concepts in the long-run.

To summarize, our analysis in Subsection 4.2 shows that in the unit-EIS case, for empirically relevant values (i.e., when $\lambda = 1.5, 2.25$ and 3 in Table 3), loss aversion, through portfolio selection, causes the LA-investor to vanish in the long run, and so does his price impact. The market selection process is excessively slow in terms of wealth shares, but in terms
of price impact, the selection mechanism is rather effective. In the following subsection, we will show that all these results carry through to cases of a non-unit EIS.

4.3 EIS\neq 1: Saving Behavior

The analysis in Subsection 4.1 suggests that when EIS is not equal to 1, loss aversion can change the investor’s saving behavior, which might affect the investor’s wealth accumulation and survival prospects. In this subsection, we investigate this possibility in the heterogeneous agent economy. Again, we assume that the preferences of both investors are identical except that \( b_{LA} > 0 \) and \( b_{EZ} = 0 \).

When \( EIS_{EZ} = EIS_{LA} > 1 \), the intuition in the representative agent economies implies that the LA-investor consumes more than the EZ-investor, which hurts his survival prospects. Because the previous subsection shows that in the absence of different saving behaviors, the LA-investor already loses to the EZ-investor; then, this extra force coming from saving should cause the LA-investor to vanish at a faster speed. This is indeed the case, as verified by Panel A of Table 6, which reports the medians of \( \omega_t \) and \( priceimpact_t \) in years 5 (\( t = 60 \)), 10 (\( t = 120 \)), and 50 (\( t = 600 \)), for the case of \( EIS_{EZ} = EIS_{LA} = 1.2 \). The technology parameters are fixed at the values in Table 1, while the other preference parameters are \( \beta_{EZ} = \beta_{LA} = 0.998 \), \( RA_{EZ} = RA_{LA} = 1 \), \( \lambda = 2.25 \) and \( b_{LA} = 0.001 \).\footnote{The results are insensitive to the choice of those parameter values, as long as the LA-investor is maintained to be more cautious in buying the stock.} Indeed, both \( \omega_t \) and \( priceimpact_t \) decline over time, suggesting that the LA-investor is losing his wealth share and price impacts in the long run. Also, the selection speed is still very slow in terms of wealth, but it is very fast in terms of price impact, and hence the market selection mechanism is very effective.

\[ \text{[INSERT TABLE 6 HERE]} \]

When \( EIS_{EZ} = EIS_{LA} < 1 \), the analysis in Subsection 4.1 suggests that the LA-investor saves more than the EZ-investor, which favors his survival. As a result, two forces are at play here: portfolio decisions are against the LA-investor’s wealth accumulation, while consumption decisions benefit it. It thus becomes nontrivial to explore whether the saving force is strong enough to reverse the result in the previous subsection. Panel B of Table
6 presents the medians of $\omega_t$ and $\text{priceimpact}_t$ for the case of $EIS_{EZ} = EIS_{LA} = 0.8$. The other parameter values are identical to those used in Panel A. We still find that all our results go through in this case, implying that in calibrated economies, the difference in saving behaviors induced by the common small EIS shared by both investors is not large enough to help the LA-investor survive and have a large price impact.

[INSERT FIGURE 2 HERE]

To better understand the effect of savings, Figure 2 plots the consumption policies for both investors and the equilibrium risk-free rate function under the parameter configuration of Panel B of Table 6. Panel (a) of Figure 2 shows that, although the difference in the endogenous monthly saving rates of the two investors can be large, with a maximum of 23 basis points (which corresponds to a difference of 2.8% in annual saving rates) achieved at $\omega_t = 1$, this difference shrinks sharply, as the LA-investor loses wealth over time. The intuition is as follows: as $\omega_t$ decreases, the EZ-investor controls more wealth, and because she saves less relative to the LA-investor, the risk-free rate increases, which in turn raises the LA-investor’s future income because he tends to invest in the risk-free rate (as suggested by Subsection 4.1) and, as a result of anticipating this rise in the permanent income, he consumes more. Then, once his wealth-eroding investment positions start to reduce his wealth share, he also saves less, making his situation even worse in terms of wealth accumulation, which explains why the difference in the endogenous saving behavior cannot overcome the disadvantage coming from the portfolio positions of the LA-investor in terms of wealth accumulation.

In sum, for the non-unit EIS case, if the LA-investor differs from the EZ-investor only in a way such that he derives loss aversion utility, then for empirically relevant parameter values, he will lose his wealth share over time, and his price impacts diminish along the way. Still, the selection speed is slow in terms of wealth, but fast in terms of price impacts.

5 Multi-Dimensional Heterogeneity in Preferences

So far, our analysis has assumed that the LA-investor and the EZ-investor are different only in one dimension: the LA-investor derives loss aversion utility, while the EZ-investor does
not. However, it is highly likely that they are also different in other dimensions. This section examines the effect additional differences in the investors’ preferences.

Before we do that, we first briefly discuss what kind of heterogeneity might be plausible in reality. In principle, on top of loss aversion utility, investors can be different in the following three dimensions: traditional risk aversion (parameter \( \zeta \)), EIS (parameter \( \rho \)), and time preference (parameter \( \beta \)). Risk aversion might not be a good candidate, as the very reason why the literature introduces loss aversion is to increase the LA-investor’s effective risk aversion, which serves to generate a high equity premium. So, to make the analysis empirically relevant, any perturbation of parameter \( \zeta \) should not reverse the order of the investors’ risk attitude and would not change the results. Researchers have not reached a consensus regarding the “reasonable” value for the EIS or the time discount rate.\(^{23}\) We therefore choose to investigate the effect of a differing EIS or a differing time-patience parameter, and we find that these parameters indeed matter dramatically for the long-run survival through affecting saving decisions.

### 5.1 Different EIS Parameters

To set the deck against the EZ-investor, we need to assume that the LA-investor has a larger EIS than the EZ-investor, so that he would save more than the EZ-investor in a growing economy. Specifically, we set \( EIS_{LA} = 0.8 \), and decrease \( EIS_{EZ} \) to see when the LA-investor will survive for an economy with the technology parameters fixed at the values in Table 1 and the other preference parameters fixed at \( \beta_{EZ} = \beta_{LA} = 0.998 \), \( RA_{EZ} = RA_{LA} = 1 \), \( \lambda = 2.25 \) and \( b_{LA} = 0.001 \). It turns out that when \( EIS_{EZ} = 0.5 \), the LA-investor starts to dominate the economy. The result is driven by the different saving behaviors induced by the different EIS. Panel (a1) of Figure 3 depicts the whole simulated probability density functions (p.d.f.s) of the LA-investor’s wealth shares assuming that \( \omega_0 = 0.5 \), where the p.d.f.s are estimated non-parametrically from the simulation data. We can see that the p.d.f.s shift to the right as

\(^{23}\)Some studies estimate the EIS to be well above 1 (e.g., Hansen and Singleton, 1982; Attanasio and Weber, 1989; Guvenen, 2001; Vissing-Jorgensen, 2002), while others estimate it to be well below 1 (e.g., Hall, 1988; Epstein and Zin, 1991; Campbell, 1999). See Guvenen (2006) for a comprehensive review of the empirical evidence on the heterogeneity in the EIS across the population. Similarly, the calibrations of the time-patience parameter \( \beta \) are widely dispersed, with its annualized counterpart \( \beta^{12} \) ranging from 0.89 (Campbell and Cochrane, 1999) to 1.1 (Brennan and Xia, 2001).
time passes, suggesting that the LA-investor tends to dominate the market in the long run. Panel (a2) shows that the difference in saving rates does not drop much when \( \omega_t \) declines from intermediate levels of \( \omega_t \), because when \( \omega_t \) decreases, the EZ-investor consumes more as a result of a strong income effect of the raised risk-free rate. Therefore, when the wealth share of the LA-investor declines due to his portfolio decisions, his advantage in terms of saving behavior will help him.

5.2 Different Time Patience Parameter \( \beta \)

We conduct a similar exercise as in examining the effect of different EIS parameters. Specifically, we set \( \beta_{LA} = 0.998 \) and decrease \( \beta_{EZ} \) to examine when the LA-investor dominates the market in the long run. The survival result is very sensitive to the time discount rate: a slight difference in \( \beta \) as small as 0.002 can overturn the effect of the LA-investor’s portfolio decisions on his survival prospects. To illustrate this sensitivity, we set \( EIS_{EZ} = EIS_{LA} = 1.2 \), which means that the deck is set against the LA-investor, as he would consume more than the EZ-investor if they had a common \( \beta \). Other preference parameters are fixed at \( RA_{EZ} = RA_{LA} = 1 \), \( \lambda = 2.25 \) and \( b_{LA} = 0.001 \), and the technology parameters are fixed at the values in Table 1. Panel (b1) of Figure 3 shows that, as time passes, the p.d.f.s of \( \omega_t \) shift to the right, suggesting that the LA-investor is accumulating wealth at a faster rate than the EZ-investor. Panel (b2) displays the large difference in the endogenous monthly saving ratios induced by the time-patience parameter. The minimum of this difference is 0.13\%, and the maximum is 0.35\%. These large magnitudes account for the LA-investor’s eventual prosperity.

The analysis in this section suggests that what matters for the long-run survival is whether investors optimize over savings decisions. This echoes the same statement made by Yan (2008) and Cvitanić and Malamud (2011). Both papers are the selection models with CRRA preferences. Our analysis sharpens their results because the recursive preference in our model can separate risk aversion from EIS, which is the right parameter determining saving decisions. So, whether the LA-investor survives depends crucially on his saving behavior and
it is an empirical issue. The recent empirical study by Tanaka, Camerer and Nguyen (2010) found that loss-averse investors also save less, which therefore suggests that they are likely to lose out in the long run.

6 Conclusion

In this paper, we construct a model to study the wealth and price implications of loss aversion in the presence of “arbitrageurs” with Epstein-Zin preferences. Our analysis shows that the market selection mechanism is effective. If loss-averse investors and arbitrageurs only differ in the way of deriving loss aversion utility, then loss-averse investors vanish and have no effect on long run asset prices for an empirically relevant range of parameters. This is because loss aversion causes investors to act as if they are even more risk averse than arbitrageurs. In the short run, although the selection process is excessively slow in terms of wealth shares, it is very effective in terms of price impacts, because the first-order risk aversion feature of loss aversion causes loss-averse investors to endogenously withdraw from the stock market. We also find that the market selects for those investors with high trading motives which can be generated by high EIS or time-patience parameters.

Our paper helps us to understand under what conditions loss aversion can affect asset prices in a dynamic financial market. Our analysis suggests that introducing heterogeneity is important for determining the price dynamics. Empirical studies are needed to examine whether and to what extent the real investors who exhibit loss aversion are different from those who do not, which in turn, with the help of our framework, is useful in further quantifying the pricing effect of preference heterogeneity. In addition, at a broad level, we believe that our findings might hold for many other types of first-order risk aversion preferences, such as ambiguity aversion and disappointment aversion. This is because the intuitions which drive our long-run and short-run results—that loss aversion raises effective risk aversion and that loss aversion implies first-order risk aversion—are shared by those other types preferences as well. We leave such a general analysis for future research.
Appendix

A. First-Order Conditions for the Case of EIS=1

This appendix derives the conditions that define the investor’s optimal decisions when the EIS takes the value of 1 ($\rho_i = 0$). In this case, the aggregator function has the Cobb-Douglas form:

$$H_i (C, X) = C^{1-\beta_i} X^{\beta_i}.$$ 

The Bellman equation becomes

$$A_{i,t} W_{i,t} = \max_{C_{i,t}, s_{i,t}} C_{i,t}^{1-\beta_i} \left[ \mu_i \left[ J_i (W_{i,t+1}, I_{t+1}) | I_t \right] + b_t E_t \left[ v \left( G_{i,t+1} \right) \right] \right]^{\beta_i}$$

$$= W_{i,t} \max_{\alpha_{i,t}, s_{i,t}} \alpha_{i,t}^{1-\beta_i} (1 - \alpha_{i,t})^{\beta_i} \left[ \mu_i \left[ A_{i,t+1} M_{i,t+1} | I_t \right] + b_t E_t \left[ v \left( s_{i,t} (R_{t+1} - R_{t,t}) \right) \right] \right]^{\beta_i}$$

Therefore, the optimal consumption policy can be explicitly solved:

$$\max_{\alpha_{i,t}} \alpha_{i,t}^{1-\beta_i} (1 - \alpha_{i,t})^{\beta_i} \Rightarrow \alpha_{i,t}^* = 1 - \beta_i. \quad (32)$$

As a result,

$$A_{i,t} = (1 - \beta_i)^{1-\beta_i} \beta_i \max_{s_{i,t}} \left[ \mu_i (A_{i,t+1} M_{i,t+1} | I_t) + b_t E_t \left[ v \left( s_{i,t} (R_{t+1} - R_{t,t}) \right) \right] \right]^{\beta_i}. \quad (33)$$

The partial equilibrium problem is therefore summarized by the above equation, which involves solving the optimal investment-decision function, $s_i (\cdot)$, and the value function, $A_i (\cdot)$. So, relative to the case of a non-unit EIS, one can avoid numerically solving the investor’s consumption policy, as it is given by equation (32), but he needs to numerically solve the investor’s indirect value function using equation (33).

The first-order conditions to the portfolio choice problem are

$$FOC_{i,+} = \left[ E_t (A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i}) \right]^{1/\zeta_i - 1} E_t \left[ A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t}) \right] + b_t E_t \left[ v (R_{t+1} - R_{f,t}) \right] = 0, \text{ for } s_{i,t}^* > 0,$$

34
\[
FOC_{i,-} = \left[ E_t(A_{i,t+1}^e M_{i,t+1}^e) \right]^{1/\xi_i-1} E_t[A_{i,t+1}^e M_{i,t+1}^e (R_{t+1} - R_{f,t})] - b_i E_t [v (R_{f,t} - R_{t+1})]
= 0, \text{ for } s_{i,t}^* < 0,
\]

\[
FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s_{i,t}^* = 0.
\]

In particular, for the EZ-investor, \( b_{EZ} = 0 \), and the above first-order conditions boil down to

\[
E_t[A_{i,EZ,t+1}^e M_{i,EZ,t+1}^e (R_{t+1} - R_{f,t})] = 0.
\]

**B. Numerical Algorithm**

This appendix sketches the procedure used to numerically solve the model. I focus on the non-unit EIS case (\( \rho_i \neq 0 \)), and the solution procedure for the unit EIS case is slightly different. The algorithm is developed based on Kubler and Schmedders (2003) and is summarized as follows.

**Step 0:** Define a finite grid on \([0,1]\). Choose two continuous functions, \( \alpha_{EZ}^0 (\cdot) \) and \( \alpha_{LA}^0 (\cdot) \), as initials for the investors’ consumption policy functions. These initials define the initial for the price-dividend ratio function, \( f^0 (\cdot) \), through equation (22). Then on each grid point \( \omega_t \), go through steps 1-4.

**Step 1:** Given functions \( \alpha_{EZ}^n (\cdot) \) and \( \alpha_{LA}^n (\cdot) \), suppose that the LA-investor allocates nothing on the stock; that is, \( s_{LA}^{n+1} (\omega_t) = 0 \). Then use both investors’ value functions, equation (13), the EZ-investor’s first-order condition, equation (17), and the state transition functions, equation (27), to solve five unknowns: \( \alpha_{EZ,t}^*, \alpha_{LA,t}^*, R_{f,t}, \omega_{t+1,H}, \omega_{t+1,L} \), where \( \omega_{t+1,H} \) and \( \omega_{t+1,L} \) are the next-period wealth shares when \( \theta_{t+1} = \theta_H \) and \( \theta_L \), respectively.

**Step 2:** Plug the solved \( \alpha_{LA,t}^*, R_{f,t}, \omega_{t+1,H} \) and \( \omega_{t+1,L} \) into equations (14) and (15) to get \( FOC_{LA,+} \) and \( FOC_{LA,-} \). If \( FOC_{LA,+} \leq 0 \text{ and } FOC_{LA,-} \geq 0 \), then set \( \alpha_{EZ,t}^{n+1} (\omega_t) = \alpha_{EZ,t}^* \) and \( \alpha_{LA}^{n+1} (\omega_t) = \alpha_{LA,t}^* \). If \( FOC_{LA,+} > 0 \), then go to Step 3; otherwise, go to Step 4.

**Step 3:** Use both investors’ value functions, equation (13), the EZ-investor’s first-order equation, (17), the LA-investor’s first-order condition for a positive investment, equation (14), and the state transition functions, equation (27), to solve six unknowns: \( \alpha_{EZ,t}^*, \alpha_{LA,t}^*, R_{f,t}, \omega_{t+1,H}, \omega_{t+1,L}, s_{LA,t}^* \). Set \( \alpha_{EZ,t}^{n+1} (\omega_t) = \alpha_{EZ,t}^* \) and \( \alpha_{LA}^{n+1} (\omega_t) = \alpha_{LA,t}^* \).
Step 4: Use both investors’ value functions, equation (13), the EZ-investor’s first-order equation, (17), the LA-investor’s first-order condition for a negative investment, equation (15), and the state transition functions, equation (27), to solve six unknowns: \( \alpha_{EZ,t}^*, \alpha_{LA,t}^*, R_{f,t}, \omega_{t+1,H}, \omega_{t+1,L}, s_{LA,t}^* \). Set \( \alpha_{EZ}^{n+1}(\omega_t) = \alpha_{EZ,t}^* \) and \( \alpha_{LA}^{n+1}(\omega_t) = \alpha_{LA,t}^* \).

Step 5: Check whether the following stop criterion is satisfied:

\[
\max_{\omega_t} \left\| \frac{(\alpha_{EZ}^{n+1}(\cdot), \alpha_{LA}^{n+1}(\cdot), f^{n+1}(\cdot))}{(\alpha_{EZ}^{n}(\cdot), \alpha_{LA}^{n}(\cdot), f^n(\cdot))} - 1 \right\| < \tau,
\]

where \( \tau \) is an error tolerance. If yes, then the algorithm terminates, and the next step is to set the consumption and investment policy functions and the risk-free rate function as those solved in the last round. Otherwise, increase \( n \) by 1 and go to Step 1.
References


This table reports the technology parameter values used in the computation of equilibria. The calibration takes one period to be one month. The consumption growth rate parameters $\theta_H$ and $\theta_L$ are calibrated to match the historical mean (1.84%) and volatility (3.79%) of the annual log consumption growth rate.
Table 2 Asset Prices and Consumption Policies in Representative Agent Economies

<table>
<thead>
<tr>
<th></th>
<th>$\bar{E}P^a$ (%)</th>
<th>$r^a_f$ (%)</th>
<th>$\alpha^a$ (%)</th>
<th>$q_L$ ($)</th>
<th>$q_S$ ($)</th>
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</thead>
<tbody>
<tr>
<td>Panel A: $EIS = 1$</td>
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<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>4.17</td>
<td>2.40</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.22</td>
<td>3.03</td>
<td>2.40</td>
<td>5076</td>
<td>21</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>2.65</td>
<td>1.59</td>
<td>2.40</td>
<td>5894</td>
<td>42</td>
</tr>
<tr>
<td>Panel B: $EIS = 0.8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>4.63</td>
<td>2.86</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.13</td>
<td>3.31</td>
<td>2.60</td>
<td>5021</td>
<td>19</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>3.01</td>
<td>0.95</td>
<td>2.13</td>
<td>6079</td>
<td>47</td>
</tr>
<tr>
<td>Panel C: $EIS = 1.2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b = 0$</td>
<td>0.07</td>
<td>3.86</td>
<td>2.09</td>
<td>4289</td>
<td>0.42</td>
</tr>
<tr>
<td>$b = 0.0005$</td>
<td>1.27</td>
<td>2.86</td>
<td>2.29</td>
<td>5111</td>
<td>22</td>
</tr>
<tr>
<td>$b = 0.001$</td>
<td>2.53</td>
<td>1.82</td>
<td>2.50</td>
<td>5832</td>
<td>41</td>
</tr>
</tbody>
</table>

This table reports the continuously compounded annualized equilibrium equity premiums ($EP^a = 12[E(\log R_{t+1}) - \log R_f]$), risk-free rates ($r^a_f = 12\log (R_f)$) and consumption propensities ($\alpha^a = 12\alpha$), assuming that investors are identical in preferences. Panels A, B and C correspond to different values of EIS: $EIS = 1$ ($\rho = 0$), $EIS = 0.8$ ($\rho = -0.25$) and $EIS = 1.2$ ($\rho = 1/6$). Parameter $b$ controls the relative importance of loss aversion utility in the investor’s preferences. For all combinations, the following three preference parameters are fixed at constant: $\beta = 0.998$, $\lambda = 2.25$ and $RA = 1$ (or $\zeta = 0$). The technology parameters are fixed at the values in Table 1. $q_L$ ($q_S$) is the premium a representative agent with wealth of $75,000 would pay to avoid a 50:50 bet to gain or lose $25,000 ($250).
Table 3 Survival of the LA-Investor: $EIS = 1$

<table>
<thead>
<tr>
<th>Panel A: $RA_{EZ} = RAL_{A} = 1 \ (\zeta_{EZ} = \zeta_{LA} = 0)$</th>
<th>$\lambda = 1.05$</th>
<th>$\lambda = 1.5$</th>
<th>$\lambda = 2.25$</th>
<th>$\lambda = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 60$</td>
<td>0.4897</td>
<td>0.4897</td>
<td>0.4897</td>
<td>0.4897</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>0.4853</td>
<td>0.4853</td>
<td>0.4853</td>
<td>0.4853</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>0.4547</td>
<td>0.4534</td>
<td>0.4534</td>
<td>0.4534</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: $RA_{EZ} = RAL_{A} = 3 \ (\zeta_{EZ} = \zeta_{LA} = -2)$</th>
<th>$\lambda = 1.05$</th>
<th>$\lambda = 1.5$</th>
<th>$\lambda = 2.25$</th>
<th>$\lambda = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 60$</td>
<td>0.5026</td>
<td>0.4810</td>
<td>0.4782</td>
<td>0.4782</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>0.5039</td>
<td>0.4667</td>
<td>0.4633</td>
<td>0.4633</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>0.5110</td>
<td>0.3702</td>
<td>0.3641</td>
<td>0.3641</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s wealth shares ($\omega_t$) at times $t = 60$, 120, and 600 months when the LA-investor has initial wealth shares of $\omega_0 = 0.5$ and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (Panel B). Both investors have a unit EIS: $EIS_{EZ} = EIS_{LA} = 1$. They have the same time patience parameter: $\beta_{EZ} = \beta_{LA} = 0.998$. In the LA-investor’s preferences, parameter $b_{LA}$ is set as 0.001, and the loss aversion coefficient $\lambda$ can take four possible values: $\lambda = 1.05, 1.5, 2.25$ and 3. The technology parameters are fixed at the values in Table 1. The medians are estimated from 5000 simulated sample paths at time $t$. 

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This table illustrates the relationship between the risk attitude and the survival prospects of the LA-investor. The variables $E_{PLA}$, $E_{Plog}$, and $E_{PEZ}$ are monthly equity premiums (in basis points) in the representative agent economies populated with the LA-investor, the log investor, and the EZ-investor, respectively. The technology parameters are fixed at the values in Table 1. The other preference parameters are set at the following values: $EIS_{EZ} = EIS_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.998$ and $RA_{EZ} = RA_{LA} = 3$. “Y” and “N” represent “Yes” and “No” respectively. The expression of “log<LA<EZ” means that in terms of risk attitude, the LA-investor behaves as if he is more risk averse than the log investor, and the EZ-investor is more risk averse than the LA-investor. A similar explanation applies to the expression of “log<EZ<LA.”
Table 5 Price Impact of the LA-Investor: $EIS = 1$

<table>
<thead>
<tr>
<th>Panel A: $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)</th>
<th>$\omega_0 = 0.1$</th>
<th>$\omega_0 = 0.5$</th>
<th>$\omega_0 = 0.9$</th>
<th>$\omega_0 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>1.1111</td>
<td>1.9995</td>
<td>9.8945</td>
<td>18.92</td>
</tr>
<tr>
<td>$t = 60$</td>
<td>1.1092</td>
<td>1.9592</td>
<td>6.7691</td>
<td>18.92</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>1.1087</td>
<td>1.9424</td>
<td>5.5034</td>
<td>18.92</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>1.1071</td>
<td>1.8290</td>
<td>2.9761</td>
<td>18.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: $RA_{EZ} = RA_{LA} = 3$ ($\zeta_{EZ} = \zeta_{LA} = -2$)</th>
<th>$\omega_0 = 0.1$</th>
<th>$\omega_0 = 0.5$</th>
<th>$\omega_0 = 0.9$</th>
<th>$\omega_0 = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>1.1093</td>
<td>1.8420</td>
<td>4.8418</td>
<td>6.6105</td>
</tr>
<tr>
<td>$t = 60$</td>
<td>1.1057</td>
<td>1.7830</td>
<td>4.1068</td>
<td>6.6105</td>
</tr>
<tr>
<td>$t = 120$</td>
<td>1.1034</td>
<td>1.7446</td>
<td>3.3067</td>
<td>6.6105</td>
</tr>
<tr>
<td>$t = 600$</td>
<td>1.0889</td>
<td>1.5204</td>
<td>1.9381</td>
<td>6.6105</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s impact on asset prices at times $t = 60$, 120, and 600 months when the LA-investor has initial wealth shares of $\omega_0 = 0.1, 0.5, 0.9$ and 1, and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). The price impact is measured by $priceimpact_t = EP_t/EP_{EZ}$, the ratio of the conditional monthly equity premium in the heterogeneous agent economy to the monthly equity premium in the economy with only the EZ-investor. Both investors have a unit EIS: $EIS_{EZ} = EIS_{LA} = 1$. They have the same time patience parameter: $\beta_{EZ} = \beta_{LA} = 0.998$. In the LA-investor’s preferences, paremeter $b_{LA}$ is set as 0.001, and the loss aversion coefficient $\lambda$ is set as 2.25. The technology parameters are fixed at the values in Table 1. The medians are estimated from 5000 simulated sample paths at time $t$. 

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Table 6 Survival and Price Impact of the LA-Investor for Non-Unit EIS

<table>
<thead>
<tr>
<th></th>
<th>Panel A: EIS=1.2</th>
<th></th>
<th>Panel B: EIS=0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \omega_0 = 0.1 )</td>
<td>( \omega_0 = 0.5 )</td>
<td>( \omega_0 = 0.9 )</td>
</tr>
<tr>
<td><strong>Medians of ( \omega_t )</strong></td>
<td>0.0984</td>
<td>0.4889</td>
<td>0.8358</td>
</tr>
<tr>
<td>( t = 60 )</td>
<td>0.0979</td>
<td>0.4838</td>
<td>0.7954</td>
</tr>
<tr>
<td><strong>Medians of ( priceimpact_t )</strong></td>
<td>0.0960</td>
<td>0.4463</td>
<td>0.6316</td>
</tr>
<tr>
<td>( t = 600 )</td>
<td>0.0986</td>
<td>0.4905</td>
<td>0.8662</td>
</tr>
</tbody>
</table>

This table reports the medians of the LA-investor’s wealth shares \( (\omega_t) \) and price impacts \( (priceimpact_t \equiv EP_t/EP_{EZ}) \) at times \( t = 60, 120, \) and 600 months when both investors have a common non-unit EIS parameter. In Panel A, \( EIS_{EZ} = EIS_{LA} = 1.2 \), and in Panel B, \( EIS_{EZ} = EIS_{LA} = 0.8 \). They have the same time patience parameter and relative risk aversion parameters: \( \beta_{EZ} = \beta_{LA} = 0.998 \) and \( RA_{EZ} = RA_{LA} = 1 \) (\( \zeta_{EZ} = \zeta_{LA} = 0 \)). In the LA-investor’s preferences, parameter \( b_{LA} \) is set as 0.001, and the loss aversion coefficient \( \lambda \) is set as 2.25. The technology parameters are fixed at the values in Table 1. The medians are estimated from 5000 simulated sample paths at time \( t \).
Figure 1 graphs the LA-investor’s price impact (priceimpact\(_t\) = \(EP_t/EP_{EZ}\)) in Panel (a) and his stock investment policy \((s_{LA,t})\) in Panel (b). The preference parameters are \(EIS_{EZ} = EIS_{LA} = 1\), \(RA_{EZ} = RA_{LA} = 1\), \(\beta_{EZ} = \beta_{LA} = 0.998\), \(\lambda = 2.25\) and \(b_{LA} = 0.001\). The technology parameters are fixed at the values in Table 1.
Figure 2 depicts the consumption policies of both investors in Panel (a) and the equilibrium risk-free rate function in Panel (b) when $EIS_{EZ} = EIS_{LA} = 0.8$. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.998$, $\lambda = 2.25$ and $b_{LA} = 0.001$. The technology parameters are fixed at the values in Table 1.
Figure 3  Survival and Consumption Policies when Investors Have Different EIS and Time Patience Parameters

Figure 3 depicts the probability density functions (p.d.f.s) of the LA-investor’s wealth shares ($\omega_t$) at $t=60,120,600$ months in Panels (a1) and (b1), as well as the consumption policies of both investors in Panels (a2) and (b2) when both investors either have different EIS parameters or time patience parameters. Specifically, in Panels (a1) and (a2), investors have different EIS parameters $EIS_{EZ}=0.5$ and $EIS_{LA}=0.8$, while they share a common time patience parameter $\beta_{EZ}=\beta_{LA}=0.998$. In Panels (b1)-(b2), they have different time patience parameters $\beta_{EZ}=0.996$ and $\beta_{LA}=0.998$, while they share a common EIS parameter $EIS_{EZ}=EIS_{LA}=1.2$. The other preference parameters are $R_{AEZ}=R_{ALA}=1$, $\lambda=2.25$ and $b_{LA}=0.001$. The technology parameters are fixed at the values in Table 1. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, $\omega_0=0.5$. 