

# Managing Opportunistic Supplier Product Adulteration: Deferred Payments, Inspection, and Combined Mechanisms

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## Abstract

Recent cases of product adulteration by foreign suppliers have compelled many manufacturers to re-think approaches to deterring suppliers from cutting corners, especially when manufacturers cannot fully monitor and control the suppliers' actions. Recognizing that process certification programs, such as ISO9000, do not guarantee unadulterated products and that product liability and product warranty with foreign suppliers are rarely enforceable, manufacturers turn to mechanisms that make payments to the suppliers contingent on no defects discovery. In this paper we study: (a) the deferred payment mechanism — the buyer pays the supplier after the deferred payment period only if no adulteration has been discovered by the customers; (b) the inspection mechanism — the buyer pays the supplier is immediately, contingent on product passing the inspection; and (c) the combined mechanism — a combination of the deferred payment and inspection mechanisms. We find the optimal contracts for each mechanism, and describe the Nash equilibria of inspection sub-games for the inspection and the combined mechanisms. The inspection mechanism cannot completely deter the suppliers from product adulteration, while the deferred payment mechanism can. Surprisingly, the combined mechanism is redundant: either the inspection or the deferred payment mechanisms perform just as well. Finally, the four factors that determine the dominance of deferred payment mechanism over the inspection mechanism are: (a) the inspection cost relative to inspection accuracy, (b) the buyer's liability for adulterated products, (c) the difference in financing rates for the buyer and the supplier relative to the defects discovery rate by customers, and (d) the difference in production costs for adulterated and unadulterated product.

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# 1 Introduction

As more firms outsource their operations to foreign suppliers, price competition among these firms intensifies and their suppliers are under tremendous pressure to cut cost. Unfortunately, in response to this pressure, many foreign suppliers reduced cost by lowering product quality and some even produced adulterated products. Product adulteration became a common cause of many recent product recalls. In 2007, public concern about product safety reached an all-time high after witnessing a record year of 472 product recalls. For example, to reduce cost, time, and potential scrutiny by the Chinese government, Baxter's contract manufacturer Changzhou SPL used cheaper and unsafe substance to produce adulterated blood-thinning drug Heparin for Baxter in 2007. Following deaths and illnesses in the US, Baxter recalled its Heparin in 2008 (Fairclough, 2008). Other recent examples include the recall of millions adulterated Mattel toys due to unapproved lead paint (Story and Barboza, 2007); the recall of a few hundred brands of adulterated pet food due to harmful ingredients such as melamine (Myers, 2007; Newman, 2007; Roth et al., 2008); and the 2008 Chinese milk scandal involving milk and infant formula adulterated with melamine triggering suspension of imports of dairy products made in China to 11 countries (Martin, 2008).

Although product recalls can be caused by product design flaws, unintended use of defective materials, and unintended contaminated production process, all aforementioned recalls were caused by the deliberate use of unapproved and harmful materials by foreign suppliers, who chose to produce adulterated products seeking higher profits. Despite product safety challenges, China continues to be an attractive place for US manufacturers to source from and a giant marketplace for US manufacturers to sell into (Fallows 2007). Thus, to improve product safety and to regain consumer confidence, many US manufacturers have implemented or refined quality management programs over different stages of the production process: (a) supplier process certification at the supplier selection stage; (b) product certification or inspection (an inspection conducted by an independent agency or by the manufacturer) after receiving product from the supplier; and (c) supplier's product liability, warranty, and (as this paper discusses) deferred payments after product is sold to the customer.<sup>1</sup>

All of the above programs have their advantages and disadvantages when used to control suppliers' incentives to produce adulterated products. For instance, to assure potential manufacturers that the supplier's production process conforms with international standards, many foreign suppliers are required to obtain process certification, such as ISO9000, to qualify as potential suppliers. Unfortunately, ISO9000 certifies only that formalized businesses processes are being established. It does not guarantee that the established processes are being followed at all times and, as such, it does not guarantee product quality when the documented process is not followed. For example, the

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<sup>1</sup>Other programs include unannounced supplier audits, business ethics training, and product safety programs.

supplier that produced lead-tainted toys for Mattel was a certified supplier who was supposed to purchase paint from Mattel’s certified vendors and to inspect the toys for lead before shipping them to Mattel.<sup>2</sup> However, for undisclosed reasons, this supplier did not comply with the documented processes, which caused Mattel to recall millions of lead-tainted toys in 2007 (Tang, 2008).

Recognizing that the supplier process certification at the supplier selection stage is not sufficient to deter product adulteration, many manufacturers establish an additional product certification process<sup>3</sup> upon receiving the product from the suppliers (e.g., an inspection can be conducted by independent agencies such as Intertek Testing Services (UK), SGS (Switzerland), or Underwriter Laboratories (USA), or by the manufacturers themselves<sup>4</sup>). For example, after a series of major recalls (caused by certified suppliers), Mattel has implemented an intensive inspection mechanism with its contract manufacturers in late 2007 (Tang, 2008). Specifically, Mattel will reject all shipments and terminate all existing contracts if a supplier did not comply with the product safety procedure. Product inspections have two major downsides for deterring product adulteration. First, they are costly. Second, inspections cannot fully guarantee product safety. For example, consider adulterated milk products produced by Chinese suppliers in 2008. Knowing that the inspection agencies will use the standard Kjeldahl and Dumas methods to test for protein levels and that these two methods fail to distinguish between nitrogen in melamine and nitrogen that naturally occurs in amino acids, these suppliers managed to falsify their products by adding harmful additive (melamine) to conceal the dilution of milk with water. Still, given the prevalence of product inspection research and practice, in this paper, we use inspection mechanism as the benchmark to evaluate the performance of other mechanisms for controlling suppliers’ incentives.

Because adulteration will be discovered by customers eventually, we focus on the post-sales programs. Specifically, we study the deferred payment mechanisms for controlling the suppliers’ incentives to adulterate products. Under the deferred payment mechanism, the buyer pays an upfront payment to initiate the production, withholds the contingent payment, and releases this contingent payment only if no adulteration is discovered by the customers over a pre-specified duration. A recent article in the Wall Street Journal (Vandenbosch and Sapp, 2010) supports the deferred payment mechanism as a way to “keep the suppliers honest.” The reasons for focusing on the deferred payment mechanism (as opposed to other post-sales programs, such as suppliers’ product liability and warranty) are twofold.

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<sup>2</sup>In many instances, manufacturers such as Mattel do not inspect the supplier’s shipments because many firms believe that once the quality process is put in place by the supplier that meets certain certification process or that follows certain guidelines, such as ISO9000, there is no need to inspect supplier shipment. See Schonberger (1986) for the case of IBM and Reimann and Hertz (1994) for the ISO9000 certification process.

<sup>3</sup>Throughout this paper, we use the term product certification and inspection interchangeably.

<sup>4</sup>While the Consumer Product Safety Improvement Act of 2008 mandates US manufacturers to submit samples children products for safety inspection to be conducted by independent laboratories, some major US manufacturers such as Mattel was allowed to conduct its inspection in-house after lobbying the new “firewall” law successfully in 2009.

First, an example of deferred payments in practice is a well-known and widely used financial contract called trade credit.<sup>5</sup> Trade credit is the largest source of external short-term financing for firms both in the US (Petersen and Rajan, 1994) and internationally (Rajan and Zingales, 1995). Trade credit is ubiquitous, especially in the current economy with banks still reluctant to lend. In our context, we build on the ideas articulated in Smith (1987), Long et al. (1993), and Lee and Stowe (1993) that deferred contingent payments (via trade credit) allow the buyers to learn about suppliers' product quality and to withhold contingent payments in case the suppliers produced defective products. Lee and Stowe (1993) argue that deferred contingent payments (via trade credit) can be thought of as an implicit product warranty and a very strong form of warranty, because the buyer can return the product to the supplier and refuse to pay without having to prove that the product is of low quality. More recently, Klapper et al. (2010) articulated that the deferred contingent payments (via trade credit) reduce the buyers' risk because the buyers have more time to investigate product quality before deciding whether or not to make the contingent payments.<sup>6</sup>

Second, other post-sales programs, such as product liability and warranties might not work well when supply chains are stretched across different continents. Specifically, when facing massive recalls of adulterated products and class action suits filed by the consumers, it is natural for the manufacturer to claim product liability from suppliers who produced adulterated products. Unfortunately, when dealing with foreign suppliers, supplier's product liability is rarely enforceable due to different legal systems and inconsistent law enforcement practices in different countries.<sup>7</sup> In some cases, the manufacturer may not even be able to trace the true identity of the fraudulent supplier.<sup>8</sup> Even if the US manufacturer wins its case against its foreign suppliers on rare occasion, most foreign suppliers rarely carry sufficient product liability insurance to settle the claim. Besides the fact that the legal process to claim supplier's product liability can take up to 10 years in foreign countries such as China, the processing cost of a legal case can be very high (Yang, 2007). In view of the reality that foreign suppliers' product liability is rarely enforceable, there is a common agreement among legal experts that it is more practical for US manufacturers to "assume" zero supplier product liability and buy their own product liability insurance with American insurance

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<sup>5</sup>For example, trade credit contract "net 30" allows the buyer to delay the payment for 30 days, giving the buyer a 30-day interest-free loan. The trade credit period depends on the industry and on the countries associated with the buyer and the supplier.

<sup>6</sup>Implicit product warranty is only one of many reasons why trade credit contracts are so popular. For a discussion of other reasons, see Petersen and Rajan (1994), Biais and Gollier (1997), Klapper et al. (2010) and references therein.

<sup>7</sup>Consider the recall of a few hundred US brands of tainted pet food that contained toxic ingredient (melamine) imported from China that killed thousands of dogs and cats in 2007. There was no way for the manufacturer to collect the liability from the Chinese supplier because the ingredient was approved by the Chinese Chief of the State Food and Drug Administration, who accepted bribes from various suppliers. As a way to settle public concerns over food safety, the Chinese decided to execute this chief quickly (Jones, 2007).

<sup>8</sup>As a way to ensure supplier traceability so that the US manufacturer can enforce supplier product liability under US federal laws, there is a pending US legislation (S.1606 Foreign Manufacturers Legal Accountability Act of 2009) that requires foreign suppliers to establish registered agents in the United States. However, this legislation is still pending as of this writing (Caldarone, 2010).

companies (Washburn and Huang, 2009; Caldarone, 2010; Rosen, 2010).

Similarly, warranties can be difficult to implement in practice. To get the supplier to pay, the buyer (and the customers) has to prove that a warranty event occurred and that the fault for this event lies with the supplier (and not with other suppliers, the buyer, or the consumer). Also, the supplier has to be financially sound at the time of the warranty event to make the warranty payment. The supplier may be unable to pay, may refuse to pay, or may declare bankruptcy, and courts may need to get involved. For example, as Sherefkin and Armstrong (2003) report, GM demanded \$37.5 million warranty payment from its supplier (Oxford Automotive Inc.) following GM's 2001 recall of Chevrolet TrailBlazer, GMC Envoy, and Oldsmobile Bravada. Because Oxford Automotive was in bankruptcy, GM and the supplier eventually settled for a smaller amount. When dealing with foreign suppliers, the issue of product warranties issued by the suppliers is even more challenging and it is more practical for the US manufacturer to issue its own "limited" product warranties (Washburn and Huang, 2009).

To summarize, in this paper we study the deferred payment mechanism for controlling the suppliers' incentive to adulterate products. We compare the performance of the deferred payment mechanism with the performance of the benchmark mechanism (i.e., inspection). Because these mechanisms are employed at different stages of the production process and each has distinct advantages, we also investigate if combining these mechanisms can generate additional benefits for the buyer.

## 1.1 Overview of the analysis and results

In §3 we present a stylized model that captures the essence of the supplier's decision to adulterate a product and the consequences of adulteration to the supplier and the buyer. Next, in §4 we discuss the deferred payment mechanism. While the deferred payment mechanism provides incentives to the supplier to produce an unadulterated product, it also may introduce an additional financing cost to the system. Specifically, the supplier has to finance its operations while the payments are deferred. If the supplier's financing cost is greater than that of the buyer (e.g., when the supplier has lower credit rating than the buyer or the supplier is located in an underdeveloped financial market), then the total financing costs of the supply chain is increased when the deferred payment mechanism is used. The additional cost of financing is eventually passed on from the supplier to the buyer through inflated wholesale prices due to the supplier's participation constraints. Furthermore, if adulterated products are sold to the customers, then the buyer is also responsible for product liability payments. We find the values of the contract parameters (upfront payment, contingent payment, and the deferred payment duration), which maximize the buyer's expected profit while assuring that the supplier produces unadulterated product and makes non-negative profit. Having found the optimal deferred payment contract, we study its properties and determine the conditions

under which this mechanism is a practical choice for the buyer.

In §5 we study the inspection mechanism. Similar to the deferred payment mechanism, the inspection mechanism provides incentives for the supplier to produce unadulterated products. However, the buyer incurs the inspection cost. Given the inspection contract parameters (upfront payment, contingent payment), we analyze the inspection sub-game (in which the supplier chooses to produce adulterated product or not and the buyer decides to inspect or not). We find that both pure and mixed strategy equilibria can occur. We find the contract that maximizes the buyer's expected equilibrium profit subject to the supplier's equilibrium profit being non-negative. For the optimal contract, it is optimal for the buyer to adopt a mixed strategy so that a random inspection of a unit or a partial inspection of a batch of products is optimal (i.e., full inspection is not optimal). The notion of random inspection has been supported by Vandenbosch and Sapp (2010) as a way to keep suppliers honest. Thus, the inspection mechanism cannot completely deter the supplier from product adulteration (while the deferred payments mechanism can).

In §6 we extend our analysis to the combined inspection and deferred payment mechanism as follows. The buyer pays an upfront payment to initiate the production, makes the first payment contingent on the inspection outcome and makes the second payment contingent on whether customers discover defects during the deferred payment period. The combined mechanism can be viewed as contingent installment payments. (The benefits of using installment payments in moral hazard situations is analyzed by Lee and Png, 1990). The second contingent payment can be interpreted as the “bonus” payment if no defects are discovered during the deferred payment period. For any given contract, we analyze the sub-game equilibria for the buyer and the supplier, determine the buyer's and the supplier's equilibrium profit, and find the optimal contract. We prove that, interestingly, the combined mechanism is redundant because either the inspection or the deferred payment mechanisms can generate the same profit for the buyer.

In §7 by comparing the deferred payment and inspection mechanisms, we identify four key factors that determine the dominance one mechanism over the other: (a) the inspection cost relative to the inspection accuracy, (b) the buyer's liability for adulterated products (c) the difference in financing rates between the buyer and the supplier relative to the rate at which customers discover defects, and (d) the difference between production costs of unadulterated and adulterated products, reflecting the supplier's incentive to cheat. We show that the deferred payment mechanism dominates when the inspection cost relative to the inspection accuracy is high, when the buyer's liability is low, or when the gap between the unadulterated and adulterated production costs is low. However, the inspection mechanism dominates when the financing cost gap relative to the rate at which customers discover defects is high. We conclude with §8.

## 2 Literature Review

At the conceptual level, our work intersects with the following research areas: supply risk, supplier quality control, and trade credit. There are many excellent review of these areas. For a general review of supply risk problems and solutions, see Tang (2006). For a more focused discussion on decentralized supply risk management, and specifically supply risk management under asymmetric information, see Aydin et al. (to appear) and Yang et al. (2009, 2008). For review of supplier quality control, see Powell (1995). Theoretical models that explain trade credit's popularity and the corresponding empirical findings are reviewed in Petersen and Rajan (1997), Biais and Gollier (1997), Ng et al. (1999), and Giannetti et al. (2008). For research on trade credit in Operations Management literature, see Baranek (1967), Haley and Higgins (1973), Rachamandugu (1989), Gupta and Wang (2009), Babich et al. (to appear), Zhou and Groenevelt (2008), Kouvelis and Zhao (2009), Yang and Birge (2009), Brunet and Babich (2007, 2009) and references therein.

We now discuss papers that are immediately related to our work. The inspection games have been studied extensively in economics, operations management, and accounting. For example, Reyniers and Tapiero (1995a,b) present models in which the supplier selects its effort in controlling product quality and the buyer determines whether to inspect or not. The buyer's contract has two components: (a) the supplier's penalty for producing a defective part; and (b) the supplier's liability for an undetected defective unit when the buyer chooses not to inspect. By analyzing a non-cooperative game, they find both mixed inspection strategy and pure inspection strategy equilibria exist for any given contract. In addition, they determine the buyer's optimal penalty structure. Recently, Baiman et al. (2000) study a model in which the supplier decides on the quality level and the buyer decides on the accuracy of the inspection. They determine the equilibrium for the quality level and the buyer's inspection accuracy level. They investigate how the inspection accuracy affects the buyer's optimal profit. Rather than dealing with the inspection accuracy, Starbird (2001) presents a model in which the buyer can select the inspection level (in terms of the size of the sample to inspect) and examines how the buyer's inspection level affects the supplier's quality level when the underlying contracts involve certain rewards and penalties associated with the supplier's quality. More recently, Chao et al. (2009) examine a situation in which the supplier and the buyer can exert their own efforts that will result in a higher quality product. To induce quality improvement efforts, they consider two types of cost sharing contracts under which each party is liable to cover certain portion of the cost associated with the root causes of product failures. By analyzing the optimal contract of each type, they show that the menu of contracts can decrease the buyer's cost and increase product quality when the supplier quality is not revealed to the buyer. Along the same vein, Balachandran and Radhakrishnan (2005) consider a similar problem; however, they focus on the use of inspection information to achieve first best quality improvement efforts

exerted by the buyer and the supplier.

In contrast to the inspection policy literature, our paper differs in the following ways. First, our paper has a different intent: we are interested in examining the extent to which various mechanisms (including inspection) deter suppliers from product adulteration. Second, we treat inspection as a benchmark mechanism, used to evaluate the performance of the deferred payment and the combined mechanisms. Third, after analyzing all three mechanisms, we derive the conditions under which one mechanism dominates the others. Fourth, relative to the inspection policy literature, our inspection model is developed in a different context: (a) our focus is on product adulteration rather than accidental variations in product quality, so that the supplier has to make a discrete decision: to produce adulterated product or not; and (b) our model reflects the fact that in practice foreign supplier's product liability is effectively unenforceable.

Next, even though the deferred payment mechanism for deterring product adulteration has not been examined before, an example of this mechanism is the financial trade credit contract. In the trade credit literature, Long et al. (1993) present an empirical model that builds on the idea articulated by Smith (1987) that trade credit can provide product quality guarantees. By analyzing a sample data that contains all industrial firms from 1984 to 1987, Long et al. (1993) provide empirical evidence to show that the trade credit period (i.e., the deferred payment period) increases when defects take more time to discover. As it turns out, our analytical results for the deferred payment model are consistent with this empirical finding. Therefore, to a certain extent, our analytical results complement the empirical work by Long et al. (1993). Emery and Nayar (1998) present a trade credit model in which the supplier knows the exact time at which the buyer can verify the quality of the product and they show that it is optimal for the supplier to demand payment at time 0 or at the instant before the buyer can verify the product quality. Similarly, Lee and Stowe (1993) propose a signalling model where the quality of the product is known to the supplier but not the buyer and find a separating equilibrium, in which trade credit terms reflect product quality. Our model differs from Smith (1987); Lee and Stowe (1993); Emery and Nayar (1998). First, we do not assume that the quality of the product is *exogenous*; instead, we consider the case when the supplier can optimally decide on whether or not to adulterate products, so that the supplier's decision is *endogenous*. Second, we allow the buyer to set the deferred payment contract terms as a way to reflect the power of large manufacturers over their (foreign) suppliers. For example, large manufacturers, such as Alcoa, specify trade credit periods (or deferred payment periods) unilaterally with its suppliers (Gamble, 2004). Thus, when analyzing the deferred contingent payment mechanism, we solve a moral hazard problem rather than a signalling problem.

In summary, our paper contributes to the literature in the following ways. First, our setting is motivated by the recent cases of adulterated products in which foreign suppliers are tempted to cut

corners by producing adulterated products deliberately. Second, recognizing that supplier’s product liability and warranty are difficult to enforce, we examine simple contingent payment mechanisms to deter suppliers from product adulteration under which the contingent payments are fully controlled by the manufacturer: deferred payment, inspection, and combined mechanisms. Third, we establish the conditions under which one mechanism dominates the others.

### 3 Models

Consider a decentralized supply chain comprising a buyer and a certified supplier (who has obtained process certification, such as ISO9000). Hence, the issue of supplier process certification, as examined in Hwang et al. (2006), is settled in advance. The buyer buys a single unit from the supplier.<sup>9</sup> The buyer is facing a moral hazard problem with respect to the supplier’s action. The supplier’s action is either  $a = n$  (i.e., produce an unadulterated (non-defective) product at a cost  $c_n$ ), or  $a = d$  (i.e., produce an adulterated (defective) product at a cost  $c_d$ ). The production cost for the adulterated product is lower,  $c_n > c_d$ , and the supplier’s action is not observable by the buyer. Because supplier is certified, it is competent, but it has an incentive to adulterate.

If the buyer decides to sell the product to the customer, the buyer receives revenue with present value  $r$  at time 0. If the product turn out to be adulterated, the defect will eventually be discovered by customers at time  $\tau$ . For tractability, we assume that  $\tau$  follows an exponential distribution with rate parameter  $\lambda$ .<sup>10</sup> When defects are discovered at time  $\tau$ , the buyer pays liability  $\rho_B$  (if  $\rho_B$  is random, we assume that it is independent from  $\tau$ ). It is convenient to define the probability that the customer will not discover the defect by time  $T$  as  $\eta(T)$ , where  $\eta(T) \stackrel{\text{def}}{=} \Pr[\tau > T] = e^{-\lambda T}$ . For ease of exposition, we suppress the explicit dependence of  $\eta$  on  $T$ , unless it is necessary. For simplicity, we assume that the supplier’s liability is zero.<sup>11</sup> Let  $\alpha_B > 0$  be the buyer’s continuously compounded financing rate. Then, the expected present value of the buyer’s product liability for selling an adulterated product equals

$$v_B \stackrel{\text{def}}{=} E[\rho_B e^{-\alpha_B \tau}] = E[\rho_B] \frac{\lambda}{\alpha_B + \lambda}. \quad (1)$$

To ensure that selling adulterated product is unprofitable for the buyer, we assume that  $v_B > r$ .

We study three contingent payment mechanisms that the buyer may use to counteract the supplier’s temptation to produce adulterated products at lower costs: the deferred payment, the

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<sup>9</sup>One can view this setting as applicable to project management where the buyer is the project manager and the supplier is the contractor. Also, the same approach can be used to analyze the case when the buyer purchases multiple units from the supplier; however, the corresponding analysis becomes tedious.

<sup>10</sup>We can obtain similar structural results when  $\tau$  follows a general distribution, where  $f(\cdot)$  is the p.d.f and  $F(\cdot)$  is the c.d.f. of this distribution, provided  $\frac{f(\cdot)}{F(\cdot)}$  is decreasing.

<sup>11</sup>The same approach can be used to analyze the case when supplier liability is strictly positive. However, as discussed in the Introduction section, supplier’s product liability is rarely enforceable in practice especially when dealing with foreign suppliers. As such, legal experts opined that foreign supplier product liability is effectively zero. When the supplier’s liability is assumed to be zero, the buyer assumes full product liability as stated in the *Consumer Product Safety Improvement Act of 2008*.

inspection, and the combined mechanisms. For any contract associated with any mechanism presented in this paper, we assume that the contract is pre-committed by all parties so that the parties cannot re-negotiate the contract later on. (In contrast, Lee and Png 1990 allow post-contract hold-up and re-negotiation that are not considered in our paper.)

### 3.1 Deferred payment

The deferred payment contract has three parts:  $(Y, q, T)$ , where  $Y \geq 0$  is the upfront payment to initiate production. Having received the product from the supplier, the buyer sells the product to the customer at time 0 and receives revenue,  $r$ . If product is adulterated, the buyer bears the product liability whose expected present value is  $v_B$ . To provide an incentive to the supplier not to adulterate the product, the buyer withholds the contingent payment  $q \geq 0$  for a duration  $T \geq 0$ , and pays the supplier at time  $T$  only if no adulteration is detected by the customer.

While waiting for the payment from the buyer, the supplier must finance its operations. Let  $\alpha_S > 0$  be the supplier's continuously compounded financing rate. The buyer and the supplier may have different financing rates. We shall focus our analysis for the case when Assumption 1 holds.

**Assumption 1.** *The supplier's financing rate is higher than the buyer's:  $\alpha_S \geq \alpha_B > 0$ .*

While our model can be easily extended to the case when Assumption 1 is violated, so that  $\alpha_S < \alpha_B$ , Assumption 1 is more likely to hold when the buyer is a larger firm with higher credit rating than the supplier. For example, Klapper et al. (2010) observe that in their data set "most suppliers are much smaller than their buyers, and are unlikely to have access to cheaper financing."

If the supplier produces an unadulterated product, the supplier pays  $c_n$  for production at time 0, receives  $Y$  at time 0, and receives  $q$  at time  $T$ . Define  $q_S \stackrel{\text{def}}{=} qe^{-\alpha_S T}$  to be the present value of the contingent payment for the supplier. The present value of the supplier's profit is  $Y + q_S - c_n$ . The corresponding present value of the buyer's profit is  $r - Y - q_B$ , where  $q_B \stackrel{\text{def}}{=} qe^{-\alpha_B T}$  is the present value of the contingent payment for the buyer. Recall that  $\eta(T) = \Pr[\tau > T]$ , is the probability that customers will not detect adulteration prior to time  $T$ . Then, if the supplier produces a defective product, the supplier pays  $c_d$  for production at time 0, receives  $Y$  at time 0, and receives  $q$  at time  $T$  with probability  $\eta$ . Thus, the expected present value of the supplier's profit is  $Y + q_S\eta - c_d$ . The corresponding expected present value of the buyer is  $r - v_B - Y - q_B\eta$ . These profits are summarized in Table 1.

|                         | Supplier            | Buyer                   |
|-------------------------|---------------------|-------------------------|
| $a = n$ (non-defective) | $Y + q_S - c_n$     | $r - Y - q_B$           |
| $a = d$ (defective)     | $Y + q_S\eta - c_d$ | $r - v_B - Y - q_B\eta$ |

**Table 1:** The supplier's and the buyer's expected discounted profits for any deferred payment contract. In these expressions  $q_S = qe^{-\alpha_S T}$  and  $q_B = qe^{-\alpha_B T}$ .

If the supplier produces an adulterated product, then the buyer's profit,  $r - v_B - Y - q_B\eta$ , is

guaranteed to be negative because (as we discussed earlier)  $r < v_B$ ,  $Y \geq 0$ , and  $q \geq 0$ . Thus, the buyer will try to induce the supplier to produce an unadulterated product with a contract that keeps  $r - Y - q_B$  positive.

### 3.2 Inspection

Under the inspection mechanism, the buyer offers a two-part inspection contract  $(X, p)$ , where  $X \geq 0$  is the upfront payment to the supplier to initiate the production of the product. (This upfront payment  $X$  can be interpreted as a deposit that is paid to the supplier prior to the inspection, if conducted.<sup>12</sup>) The second payment  $p \geq 0$  is contingent on whether a defect is detected by the inspection at time 0<sup>13</sup>.

Because we are using inspection mechanism as a benchmark, we purposefully consider a simple but salient inspection model. Specifically, given any two-part inspection contract  $(X, p)$ , the supplier and the buyer engage in an inspection sub-game, where the pure strategies of the supplier are to produce non-defective ( $a = n$ ) or defective ( $a = d$ ) product and the pure strategies of the buyer are to inspect ( $i = 1$ ) or not to inspect ( $i = 0$ ) the product. The inspection can be conducted either by the buyer directly or by a third party laboratory, and the test results are known and verifiable by all parties. The cost of inspection to the buyer is  $I > 0$ .

If the supplier delivered an unadulterated (non-defective) product, then no defects will be discovered by the inspection. That is,  $\Pr[\text{inspection report} = \text{non-defective} | a = n] = 1$ . However, if the supplier delivered an adulterated (defective) product, then the inspection sends a correct signal with probability  $\mu > 0$  so that  $\Pr[\text{inspection report} = \text{defective} | a = d] = \mu$ . Hence,  $\mu$  measures the “accuracy” of the inspection. If the inspection indicates that the product is defective, then the buyer rejects the product without paying the contingent payment  $p$ .

However, if the inspection indicates that the product is non-defective, then the product is indeed unadulterated and the buyer must (legally) accept the product and pays the contingent payment  $p$  to the supplier.<sup>14</sup> Also, in the event when the buyer decides not to inspect the product, the buyer has no new information about the product. Hence, the buyer must also (legally) accept the product and pays the contingent payment  $p$  to the supplier. In summary, if no defect is discovered during inspection (or due to no inspection), then the buyer presumes the product is non-defective, accepts the product, pays the contingent payment  $p$  to the supplier, sells the product to a customer, and receives revenue with present value  $r$  at time 0. Recall that the expected present values of liabilities for the adulterated product is  $v_B$ .

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<sup>12</sup>Clearly, the incentive for the supplier to produce adulterated products drops significantly if one allows  $X < 0$ , so that it is the supplier who pays the buyer as a “bond” to ensure product quality. However, this case is not realistic when foreign suppliers are usually cash strapped, which preclude them from offering bonds to the buyers.

<sup>13</sup>This is an approximation of the fact that inspection takes a short time relative to the deferred payment duration.

<sup>14</sup>Besides the fact that the inspection result can be verified by the supplier, there is no incentive for the buyer to falsify the inspection result and return non-defective product by incurring additional handling and shipping cost.

Table 2 presents the strategic form of the inspection sub-game. The top expressions in each cell is the expected discounted profit of the supplier and the bottom expression is the expected discounted profit of the buyer. For example, if the supplier chooses to produce an adulterated (defective) product by setting  $a = d$  and the buyer chooses not to inspect by selecting  $i = 0$ , then the supplier's discounted profit is equal to  $X + p - c_d$  and the buyer's discounted profit is equal to  $r - v_B - X - p$ .

|                         | $i = 0$ (do not inspect)              | $i = 1$ (inspect)                                             |
|-------------------------|---------------------------------------|---------------------------------------------------------------|
| $a = n$ (non-defective) | $(X + p - c_n,$<br>$r - X - p)$       | $(X + p - c_n,$<br>$r - X - p - I)$                           |
| $a = d$ (defective)     | $(X + p - c_d,$<br>$r - v_B - X - p)$ | $(X + p(1 - \mu) - c_d,$<br>$(r - p - v_B)(1 - \mu) - X - I)$ |

**Table 2:** The supplier's (top expression) and the buyer's (bottom expression) expected discounted profits for any combination of pure strategies selected by the players under the inspection contract  $(X, p)$ .

To simplify our exposition and to eliminate trivial cases, we make two mild assumptions. These assumptions can be relaxed, but the resulting analysis will become more tedious because the number of cases that needs to be analyzed increases significantly. As we shall see, these two assumptions enable us to obtain closed form solutions and to compare the optimal contract outcomes in equilibrium under different contingent payment mechanisms.

**Assumption 2.** *The inspection is accurate: when inspection is conducted, the supplier's expected penalty for getting caught is higher than the supplier's expected gain from cutting corners. Because the defect is detected by the buyer with probability  $\mu$ , this assumption implies that  $\mu c_d > (1 - \mu)(c_n - c_d)$  or equivalently,  $\mu$  is sufficiently high so that  $\mu > \frac{c_n - c_d}{c_n}$ .*

Let us consider the case when Assumption 2 is violated so that the inspection is less accurate:  $\mu \leq \frac{c_n - c_d}{c_n}$ . In this case, it is easy to check from Table 2 that, regardless of the buyer's inspection decision, the supplier will always choose to produce adulterated products as long as  $p \leq c_n$ . Therefore, when Assumption 2 is violated, the supplier's dominant strategy is to produce adulterated products when  $p \leq c_n$ . Hence, Assumption 2 enables us to eliminate this trivial case.

**Assumption 3.** *The inspection is cost effective: when inspection is conducted, the inspection cost,  $I$ , is low enough so that inspection will reduce the buyer's risk in the event when the supplier chooses to produce a defective product:  $I/\mu < v_B - r$ .*

Let us consider the case when Assumption 3 is violated so that the inspection is not cost effective, say,  $I/\mu > v_B - r$ . In this case, one can check from Table 2 that, when the contingent payment is sufficiently low so that  $p \leq I/\mu - (v_B - r)$ , the buyer will never inspect regardless of the supplier's action. This is because  $r - X - p - I \leq r - X - p$  and  $(r - p - v_B)(1 - \mu) - X - I \leq r - v_B - X - p$ . Hence, Assumption 3 enables us to eliminate this trivial case.

### 3.3 Combined Inspection and Deferred Payment

The sequence of events associated with the combined mechanism is as follows. The buyer offers a contract  $(X, p, q, T)$  to the supplier, where  $X \geq 0$  is the upfront payment and  $p$  is the payment at time 0 contingent on the inspection not discovering any defects. If no defects were discovered, the product is sold to the customers. In this case,  $q$  is the second contingent payment to the supplier if the customer does not discover adulteration by time  $T$ , where  $T$  is the deferred payment duration.

Because the combined mechanism “combines” the inspection and the deferred payment mechanism, we can use the supplier’s and the buyer’s payoffs under both mechanisms as reported in Tables 1 and 2 to determine the supplier’s and the buyer’s profit under the combined mechanism as reported in Table 3. Recall that  $q_S = e^{-\alpha_S T} q$ , and  $q_B = e^{-\alpha_B T} q$ . For example, consider the case when the supplier produces defective product ( $a = d$ ) and the buyer inspects ( $i = 1$ ). By noting that the supplier receives the upfront payment  $X$ , incurs the production cost  $c_d$ , receives the first contingent payment  $p$  with probability  $(1 - \mu)$  at time 0 (i.e., the probability that the inspection fails to report adulteration at time 0), and receives the second contingent payment  $q$  at time  $T$  with probability  $(1 - \mu)\eta$  (i.e., the probability that the inspection fails to report adulteration multiplied by the probability that the customer fails to discover adulteration by time  $T$ ), we conclude that the supplier’s payoff under this pair of pure strategies ( $a = d, i = 1$ ) equals  $X - c_d + (1 - \mu)p + (1 - \mu)\eta q_S$ . Similarly, we determine the buyer’s payoff under this pair of pure strategies.

|                         | $i = 0$ (do not inspect)                                    | $i = 1$ (inspect)                                                                     |
|-------------------------|-------------------------------------------------------------|---------------------------------------------------------------------------------------|
| $a = n$ (non-defective) | $(X + p + q_S - c_n,$<br>$r - X - p - q_B)$                 | $(X + p + q_S - c_n,$<br>$r - X - p - q_B - I)$                                       |
| $a = d$ (defective)     | $(X + p + q_S \eta - c_d,$<br>$r - v_B - X - p - q_B \eta)$ | $(X + (p + q_S \eta)(1 - \mu) - c_d,$<br>$(r - v_B - p - q_B \eta)(1 - \mu) - X - I)$ |

**Table 3:** The supplier’s (top expression) and the buyer’s (bottom expression) expected discounted profits for any combination of pure strategies selected by the players under the combined inspection and deferred payment contract  $(X, p, q, T)$ .

## 4 Analysis of the Deferred Payment Mechanism

We now analyze the deferred payment mechanism described in §3.1. Our analysis is based on a Stackelberg game in which the buyer is the leader who specifies the deferred payment contract  $(Y, q, T)$  and the supplier is the follower who decides on her product quality action ( $a \in \{n, d\}$ ). Under the deferred payment contract, the buyer pays an upfront payment  $Y \geq 0$  to the supplier at time 0 to initiate production. However, the buyer withholds the contingent payment  $q \geq 0$ , and releases this contingent payment only if no adulteration is discovered by the customer by time  $T$ . By considering the buyer’s and the supplier’s payoffs given in Table 1 for any deferred payment contract  $(Y, q, T)$ , we determine the optimal contract  $(Y^*, q^*, T^*)$  in equilibrium. We now

show that the optimal deferred payment contract provides enough incentive to prevent the supplier from product adulteration. Also, we derive conditions under which the buyer extracts the entire surplus from the supplier. Hence, the deferred payment mechanism can be an effective approach for deterring supplier from product adulteration.

#### 4.1 The Buyer's Problem under the Deferred Payment Mechanism

Recall from our discussion in §3.1 that the buyer will always prefer selling the non-defective product for any deferred payment contract  $(Y, q, T)$ . For this reason, the buyer would like to determine an optimal deferred payment contract that would entice the supplier to produce unadulterated products by choosing  $a = n$ . For any deferred payment contract  $(Y, q, T)$ , let  $\pi_S^a(Y, q, T)$  be the supplier's expected profit and let  $\pi_B^a(Y, q, T)$  be the buyer's expected profit for any supplier's action  $a$ , where  $a \in \{n, d\}$ .

To entice the supplier to produce unadulterated product in a rational manner, the deferred payment contract  $(Y, q, T)$  must satisfy: (a) the supplier's incentive compatibility constraint  $\pi_S^n(Y, q, T) \geq \pi_S^d(Y, q, T)$ ; and (b) the supplier's individual rationality constraint  $\pi_S^n(Y, q, T) \geq 0$ . By considering the payoffs presented in Table 1, these two constraints can be rewritten as:

$$q_S(1 - \eta) \geq c_n - c_d, \text{ and} \quad (2a)$$

$$Y + q_S - c_n \geq 0, \quad (2b)$$

where  $q_S = qe^{-\alpha_S T}$  is the present value of the contingent payment from the supplier's perspective. When a deferred payment contract  $(Y, q, T)$  satisfies the above constraints, the supplier will produce unadulterated products by taking the action  $a = n$ . Consequently, the buyer will obtain an expected profit  $\pi_B^n(Y, q, T)$ . By using the buyer's profit  $\pi_B^n(Y, q, T)$  presented in Table 1 along with the above constraints, we can formulate the buyer's problem as:

$$\max_{Y, q, T} r - Y - q_B \quad (3a)$$

$$\text{s.t. } q_S(1 - \eta) \geq c_n - c_d, \quad (3b)$$

$$Y + q_S \geq c_n, \quad (3c)$$

$$Y \geq 0, \quad q \geq 0, \text{ and } T \geq 0, \quad (3d)$$

where  $q_B = qe^{-\alpha_B T}$  is the present value of the contingent payment from the buyer's perspective.

#### 4.2 Optimal Deferred Payment Contract

To determine the optimal deferred payment contract, we solve problem (3) by transforming our decision variables from  $(Y, q, T)$  to  $(Y, q_S, T)$ , where  $q_S = qe^{-\alpha_S T}$ . Consequently, the buyer's problem (3) becomes

$$\max_{Y, q_S, T} r - Y - q_S e^{(\alpha_S - \alpha_B)T} \quad (4a)$$

$$\text{s.t. } q_S(1 - \eta) \geq c_n - c_d, \quad (4b)$$

$$Y + q_S \geq c_n, \quad (4c)$$

$$Y \geq 0, \quad q_S \geq 0, \quad T \geq 0. \quad (4d)$$

The solution to problem (4) is presented in the following proposition.

**Proposition 1.** *The following is the solution of the buyer's problem (4).*

1. If  $\frac{c_d}{c_n} \geq \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$ , then the optimal deferred payment contract  $(Y^*, q_S^*, T^*)$  satisfies

$$Y^* = 0, \quad q_S^* = c_n, \quad \text{and} \quad e^{-\lambda T^*} = \frac{c_d}{c_n}. \quad (5)$$

The corresponding buyer's profit is  $r - c_n \left(\frac{c_d}{c_n}\right)^{-\frac{\alpha_S - \alpha_B}{\lambda}}$ . The supplier's profit is 0.

2. If  $\frac{c_d}{c_n} < \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$ , then the optimal deferred payment contract  $(Y^*, q_S^*, T^*)$  satisfies

$$Y^* = 0, \quad q_S^* = \frac{(c_n - c_d)(\lambda + \alpha_S - \alpha_B)}{\lambda} \geq c_n, \quad \text{and} \quad e^{-\lambda T^*} = \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}. \quad (6)$$

The corresponding buyer's profit is  $r - (c_n - c_d) \frac{\lambda + \alpha_S - \alpha_B}{\lambda} \left(\frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}\right)^{-\frac{\alpha_S - \alpha_B}{\lambda}}$ . The supplier's profit is  $(c_n - c_d) \frac{\lambda + \alpha_S - \alpha_B}{\lambda} - c_n$ .

The proofs of all lemmas and propositions are presented in the Appendix.

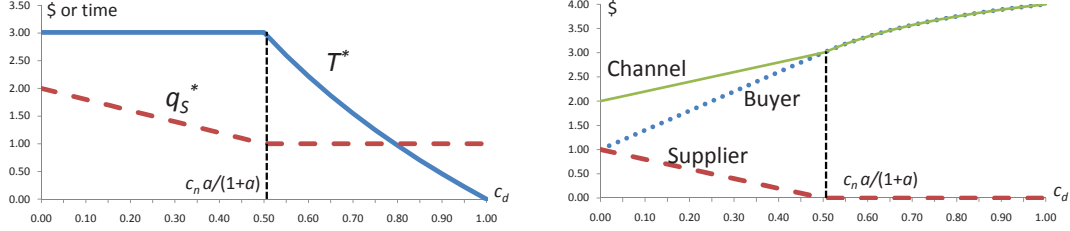
### 4.3 Properties of the Optimal Deferred Payment Contract

Proposition 1 reveals that the buyer will always offer an “effective” deferred payment  $q_S^* \geq c_n$  that deters the supplier from adulteration entirely. In fact, regardless which part of Proposition 1 holds, the incentive compatibility constraint  $q_S(1 - \eta) \geq c_n - c_d$  and the upfront payment constraint  $Y \geq 0$  are binding under the optimal contract. However, the individual rationality constraint  $Y + q_S - c_n \geq 0$  is binding in part 1 and not binding in part 2 of Proposition 1.

We now examine the comparative statics of different quantities presented in Proposition 1 with respect to the cost of producing an adulterated product  $c_d$ . For ease of exposition, we shall describe our results graphically in Figure 1 even though we can prove these comparative statics analytically.

First, consider the case when  $c_d \approx c_n$ , so that  $\frac{c_d}{c_n} \geq \frac{a}{1+a}$ , where  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . The interpretation of this case is that the supplier's incentive to adulterate the product is not very high. In this case, we can check from part 1 of Proposition 1 that the “effective” deferred contingent payment  $q_S$  is fixed ( $q_S^* = c_n$ ) and the buyer can deter adulteration by extending the optimal deferred payment contract duration,  $T^*$  as  $c_d$  decreases. As depicted in Figure 1, the supplier makes zero profit and the buyer's profit is fairly close to that of the centralized system ( $r - c_n = 4$ ).

Second, consider the case when  $c_d \ll c_n$  so that  $\frac{c_d}{c_n} < \frac{a}{1+a}$ . The interpretation of this case is that the supplier's incentive to adulterate the product is very high. In this case, as part 2



**Figure 1:** Deferred payment optimal contract, comparative statics with respect to  $c_d$ .

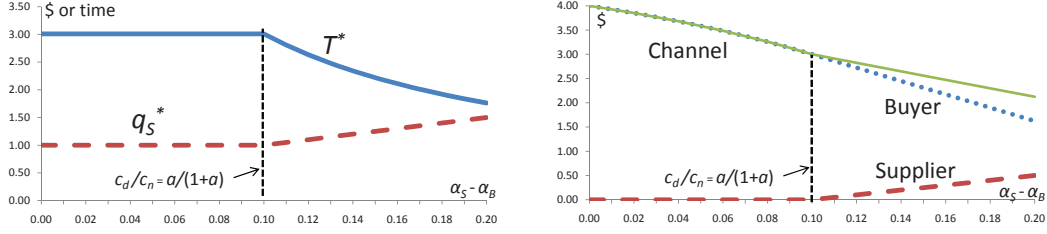
**Legend:** Left panel: optimal  $T^*$  and  $q_S^*$ . Right panel: optimal buyer's, supplier's, and channel profits. Parameter values:  $c_n = 1$ ,  $r = 5$ ,  $\alpha_S = 0.2$ ,  $\alpha_B = 0.1$ ,  $\lambda = 0.1$ ,  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . Centralized system profit is  $r - c_n = 4$ .

of Proposition 1 indicates, to prevent adulteration it is no longer sufficient to pay the supplier the minimum contingent payment ( $q_S^* = c_n$ ). Instead, the optimal deferred contingent payment  $q_S^* > c_n$  increases as  $c_d$  decreases, while the optimal duration  $T^*$  remains fixed. The supplier's profit is positive and the system overall deviates significantly from the centralized system's profit.

According to Proposition 1, the optimal deferred payment duration,  $T^*$ , is capped by the value in part 2. This facilitates the use of the deferred payment contract in practice. For part 2, the cost gap,  $c_n - c_d$ , is significant and the optimal duration is  $T^* = \frac{1}{\lambda} \ln \left( 1 + \frac{\lambda}{\alpha_S - \alpha_B} \right)$ . Thus, the optimal duration equals the average time it takes to discover defects,  $\frac{1}{\lambda}$ , adjusted by the coefficient  $\ln \left( 1 + \frac{\lambda}{\alpha_S - \alpha_B} \right)$  for financing costs. Therefore,  $T^* > \frac{1}{\lambda}$  if and only if  $\frac{\lambda}{\alpha_S - \alpha_B} > e - 1$ . For part 1 the cost gap,  $c_n - c_d$ , is small and the optimal duration is  $T^* = \frac{1}{\lambda} \ln \left( \frac{c_n}{c_d} \right)$ . Thus, the optimal duration equals the average time for the customer to discover defects adjusted by the coefficient  $\ln \left( \frac{c_n}{c_d} \right)$ . The cost ratio  $\frac{c_n}{c_d}$  has to be greater than  $e \approx 2.72$  for the optimal deferred payment duration to exceed the average time of defect discovery.

To see that the model prediction are reasonable in practice, consider the following values of the parameters: average time to discover defects  $\frac{1}{\lambda} = \frac{1}{4}$  years = 3 months, the supplier's financing rate  $\alpha_S = 20\%$  per year, the buyer's financing rate  $\alpha_B = 7\%$  per year. Then the cap on the optimal duration of the deferred payment contract is 0.86 years  $\approx 10.38$  months (from part 2 of Proposition 1). A deferred payment duration 10 months is high in comparison to the average duration of trade credit contracts in the US. However, it is feasible for trade credit contracts in countries with less developed financial systems. If the cost gap,  $c_n - c_d$ , is small, then the optimal deferred payment duration is even smaller (given in part 1 of Proposition 1). For instance, suppose that  $\frac{c_d}{c_n} = 1/2$ , then the optimal deferred payment duration is 0.17 years  $\approx 2.08$  months, which is a feasible trade credit duration even in the US. It is interesting to ask how large the cost gap can be before a 30-day trade credit contract stops being an effective in controlling the supplier's incentives. The answer is that as long as  $\frac{c_d}{c_n} > 71.65\%$ , a 30-day trade credit contract gives sufficient incentives for the supplier not to adulterate the product.

Interestingly, the inefficiencies of the deferred payment mechanism are due to two factors: cost gap in producing unadulterated and adulterated products (which creates incentives for the suppliers to cheat), and financing rate gap (which makes it expensive for the system to use deferred payments to control the supplier's incentives to cheat). As we can see from Figure 1, for fixed financing gap,  $\alpha_S - \alpha_B$ , the smaller the cost gap,  $c_n - c_d$ , the closer the channel profit to the centralized system profit,  $r - c_n$ . On the other hand, the smaller the financing gap,  $\alpha_S - \alpha_B$ , the closer the channel profit to the profit of the centralized system. In particular, for small  $\alpha_S - \alpha_B$ , part 1 of Proposition 1 applies and the channel profit is  $r - c_n \left( \frac{c_d}{c_n} \right)^{-\frac{\alpha_S - \alpha_B}{\lambda}}$ , which converges to the centralized system profit  $r - c_n$ , as  $\alpha_S - \alpha_B \rightarrow 0$ . Figure 2 illustrates this observation.



**Figure 2:** Deferred payment optimal contract, comparative statics with respect to  $\alpha_S - \alpha_B$ .

**Legend:** Left panel: Optimal  $T^*$  and  $q_S^*$ . Right panel: Optimal buyer's, supplier's, and channel profits. Parameter values:  $c_n = 1$ ,  $c_d = 0.5$ ,  $r = 5$ ,  $\lambda = 0.1$ ,  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . The centralized system profit is  $r - c_n = 4$ .

Among other takeaways from Proposition 1, the duration of the optimal deferred payment contract  $T^*$  is decreasing in the rate of defect discovery,  $\lambda$ . As it turns out, this result has been confirmed empirically by Long et al. (1993). Therefore, our analytical result complements the empirical result obtained by Long et al. (1993).

## 5 Analysis of the Inspection Mechanism

To analyze the inspection mechanism as a benchmark, we first determine the supplier's action and the buyer's inspection decision in equilibrium for any given inspection contract  $(X, p)$ . Then we find the optimal contract  $(X^*, p^*)$ . As we shall see, it is optimal for the buyer not to offer any upfront payment (i.e.,  $X^* = 0$ ), and, under certain conditions, the buyer can extract the entire surplus from the supplier. Moreover, the optimal inspection contract cannot completely deter the supplier from product adulteration, and it cannot completely eliminate the need for the buyer to conduct inspection.

### 5.1 Analysis of the Inspection Game

For any given contract  $(X, p)$ , we now analyze a non-cooperative game in which the supplier decides whether to adulterate and the buyer chooses whether to inspect. We shall refer to this game as the inspection game. The solution of the inspection game is presented in Lemma 1. In preparation, let us define the supplier's "adulteration" probability as  $x \stackrel{\text{def}}{=} \Pr[a = d]$ , and the buyer's "inspection"

probability as  $y \stackrel{\text{def}}{=} \Pr[i = 1]$ .

**Lemma 1.** *For any given inspection contract  $(X, p) \geq 0$ , the equilibrium of the inspection game can be described as follows.*

1. *If  $p < \frac{c_n - c_d}{\mu}$ , then the pure strategy  $(x^* = 1, y^* = 1)$  is a unique equilibrium. The corresponding equilibrium profits are*

$$\pi_S(X, p) = X + p(1 - \mu) - c_d, \quad (7a)$$

$$\pi_B(X, p) = -X - I - (v_B - r + p)(1 - \mu). \quad (7b)$$

2. *If  $p > \frac{c_n - c_d}{\mu}$ , then the inspection game has a unique mixed strategy equilibrium:  $x^* = \frac{I}{\mu(p - r + v_B)}$  and  $y^* = \frac{c_n - c_d}{p\mu}$ , where  $x^* \in (0, 1)$  and  $y^* \in (0, 1)$ . The supplier's and the buyer's profits are*

$$\pi_S(X, p) = X + p - c_n, \quad (8a)$$

$$\pi_B(X, p) = -X + r - p - \frac{Iv_B}{(p - r + v_B)\mu}. \quad (8b)$$

3. *If  $p = \frac{c_n - c_d}{\mu}$ , then any point  $(x, 1)$ , where  $x \in \left[\frac{I}{\mu(p - r + v_B)}, 1\right]$ , is an equilibrium of the inspection game. The supplier's and the buyer's equilibrium profits are*

$$\pi_S(X, p) = X + p - c_n, \quad (9a)$$

$$\pi_B(X, p) = -X - I + (r - p)(1 - x\mu) - xv_B(1 - \mu). \quad (9b)$$

Lemma 1 has the following implications. First, consider part 1 when  $\mu p < c_n - c_d$  so that the expected loss of contingent payment for the supplier is less than the gain from adulteration. In this case, the supplier will definitely cheat by using adulterated materials. At the same time, the buyer needs to conduct 100% inspection to reduce the risk of selling adulterated product to the customer. This pure strategy is consistent with Mattel's inspection policy after recalling millions of lead-tainted toys in 2007: Mattel is conducting inspection of lead for every single batch of toys produced by the supplier (Tang, 2008).

Second, consider part 2 when  $\mu p > c_n - c_d$  so that the expected loss of contingent payment for the supplier is greater than the gain from adulteration. In this case, the supplier would like to cheat, however, to avoid getting caught, the supplier does not always cheat and the buyer does not always inspect, so that  $x^* \in (0, 1)$  and  $y^* \in (0, 1)$ . Observe that  $x^* > 0$  even when the payment  $p$  is very large. Hence, there is always a chance for the supplier to adulterate. This observation suggests that the inspection mechanism cannot completely deter the supplier from product adulteration. The same conclusion applies even if the inspection were perfectly accurate,  $\mu = 1$ . This observation suggests that under the inspection mechanism there is always a possibility that the supplier will adulterate and that the adulterated product will reach the customer.

Finally, when  $p = \frac{c_n - c_d}{\mu}$ , as in part 3 of Lemma 1, the inspection game has multiple equilibria  $(x, 1)$ , where  $x \in \left[ \frac{I}{\mu(p-r+v_B)}, 1 \right]$ . However, by noting that the buyer's profit  $\pi_B(X, p)$  decreases in the supplier's adulteration probability  $x$  and the supplier's profit  $\pi_S(X, p)$  is independent of  $x$ , we can focus on the smallest  $x$  as the "payoff dominant equilibrium" as stated in the following corollary.

**Corollary 1.** *If  $p = \frac{c_n - c_d}{\mu}$ , then the inspection game has a unique mixed strategy payoff dominant equilibrium:  $x^* = \frac{I}{\mu(p-r+v_B)}$  and  $y^* = 1$ , where  $x^* \in (0, 1)$ . The supplier's and the buyer's profits are given in (8).*

## 5.2 Optimal Inspection Contract

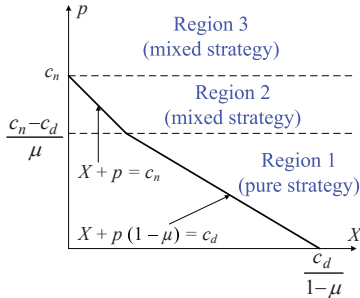
For any given contract  $(X, p)$ , the supplier's and the buyer's payoffs in equilibrium are stated in Lemma 1. We now use these payoffs to determine the optimal inspection contract by solving the buyer's problem that can be formulated as the following mathematical program:

$$\max \quad \pi_B(X, p) \quad (10a)$$

$$\text{s.t.} \quad \pi_S(X, p) \geq 0, \quad (10b)$$

$$X \geq 0, \text{ and } p \geq 0. \quad (10c)$$

In this program, (10b) is the standard supplier's participation constraint. In preparation, it is useful to use the graph the feasible region associated with the decision variables  $(X, p)$  associated with the buyer's problem (10). By using Assumption 2 along with expressions (7) and (8), it can be shown that the feasible region for the decision variables  $(X, p)$  can be divided into three regions (see Figure 3). Region 1 (i.e.,  $p < \frac{c_n - c_d}{\mu}$ ) corresponds to the equilibrium pure strategy ( $a = d, i = 1$ ), where the supplier's and the buyer's profit are given in (7). Regions 2 and 3 (i.e.,  $p \geq \frac{c_n - c_d}{\mu}$ ) correspond to the equilibrium mixed strategy  $(x^*, y^*)$ , where the supplier's and the buyer's profit are given in (8). By determining the optimal solution of the buyer's problem (10) for each of these



**Figure 3:** Feasible regions for the buyer's problem under the inspection mechanism

three feasible regions, we establish the following Lemma:

**Lemma 2.** *The optimal inspection contract associated with the feasible regions depicted in Figure 3 can be described as follows:*

1. When  $0 \leq p < \frac{c_n - c_d}{\mu}$ , the optimal inspection contract in region 1 satisfies  $X^{(1)} = c_d - p^{(1)}(1 - \mu)$  for any  $p^{(1)} \in [0, \frac{c_n - c_d}{\mu}]$ . The supplier's profit  $\pi_S^{(1)}(X^{(1)}, p^{(1)}) = 0$  and the buyer's profit

$$\pi_B^{(1)}(X^{(1)}, p^{(1)}) = -c_d - I - (v_B - r)(1 - \mu) < 0. \quad (11)$$

2. When  $p \geq \frac{c_n - c_d}{\mu}$ , the optimal inspection contract associated with the mixed strategy  $(x^*, y^*)$  in regions 2 and 3 satisfies  $X^{(2)} = 0$  and  $p^{(2)} = \max\{c_n, p^0\}$ , where  $p^0 \stackrel{\text{def}}{=} r - v_B + \sqrt{\frac{Iv_B}{\mu}}$ . The supplier's profit  $\pi_S^{(2)}(X^{(2)}, p^{(2)}) = p^{(2)} - c_n$ , and the buyer's profit satisfies:

$$\pi_B^{(2)}(X^{(2)}, p^{(2)}) = r - p^{(2)} - \frac{Iv_B}{(p^{(2)} - r + v_B)\mu}. \quad (12)$$

Lemma 2 characterizes the optimal inspection contract in each of the three regions as depicted in Figure 3. To determine the global optimal inspection contract  $(X^*, p^*)$ , we now compare the buyer's profits  $\pi_B^{(1)}(X^{(1)}, p^{(1)})$  and  $\pi_B^{(2)}(X^{(2)}, p^{(2)})$  given in (11) and (12), respectively. Knowing that the buyer's optimal profit  $\pi_B^{(1)}(X^{(1)}, p^{(1)})$  given in (11) is negative, the buyer should not participate in this contract if its optimal profit  $\pi_B^{(2)}(X^{(2)}, p^{(2)})$  is also negative. By examining the conditions under which the buyer's profit  $\pi_B^{(2)}(X^{(2)}, p^{(2)})$  is positive, we obtain the following results:

**Proposition 2.** *The optimal inspection contract, the buyer's optimal payoff, and the supplier's optimal payoff associated with the inspection game can be described as follows:*

1. If  $\frac{I}{\mu} > \frac{v_B}{4}$ , then the buyer should not participate in the Inspection mechanism.
2. If  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ , then the optimal inspection contract  $(X^*, p^*) = (0, p^0)$ , where  $p^0 = r - v_B + \sqrt{\frac{Iv_B}{\mu}} \geq c_n$ , the buyer's and the supplier's optimal profits are

$$\pi_B(X^*, p^*) = v_B - 2\sqrt{\frac{Iv_B}{\mu}} > 0, \quad (13a)$$

$$\pi_S(X^*, p^*) = p^0 - c_n > 0. \quad (13b)$$

The equilibrium probabilities are

$$x^* = \sqrt{\frac{I}{\mu v_B}}, \quad y^* = \frac{c_n - c_d}{p^0 \mu}. \quad (14)$$

3. If  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $v_B - \sqrt{\frac{Iv_B}{\mu}} \geq r - c_n \geq \frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ , then the optimal inspection contract  $(X^*, p^*) = (0, c_n)$ , the buyer's and the supplier's optimal profits are

$$\pi_B(X^*, p^*) = r - c_n - \frac{Iv_B}{(c_n + v_B - r)\mu} > 0, \quad (15a)$$

$$\pi_S(X^*, p^*) = 0. \quad (15b)$$

The equilibrium probabilities are

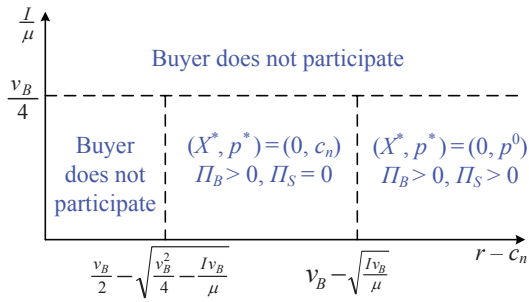
$$x^* = \frac{I}{(c_n + v_B - r)\mu}, \quad y^* = \frac{c_n - c_d}{c_n \mu}. \quad (16)$$

4. If  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n < \frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ , then the buyer should not participate in the Inspection mechanism.

### 5.3 Properties of the Optimal Inspection Contract

It is interesting to observe from parts 2 and 3 of Proposition 2 that, in equilibrium under the optimal contract, only the buyer's optimal inspection probability  $y^*$  depends on the supplier's cost for producing adulterated product  $c_d$ . Knowing that the supplier has more incentive to cheat when  $c_d$  is low, this result can be interpreted intuitively as follows. Under the inspection mechanism, it is the buyer's inspection probability  $y^*$  that keeps the supplier in check. When  $c_d$  is low, the supplier has more incentive to cheat. However, the supplier will not increase her adulteration probability  $x^*$  because the supplier anticipates the buyer would increase its inspection probability  $y^*$ . This explains why the supplier's adulteration probability  $x^*$  and the supplier's optimal profit  $\pi_S(X^*, p^*)$  are independent of  $c_d$ . Therefore, the buyer's optimal profit  $\pi_B(X^*, p^*)$  is also independent of  $c_d$ .

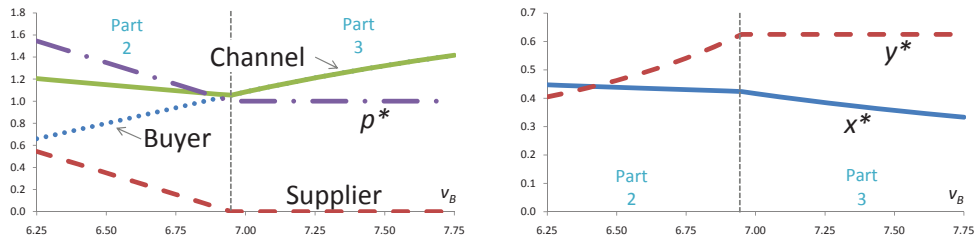
From Proposition 2, the buyer's optimal inspection contract and the buyer's participation depend on two key factors: the inspection cost to inspection accuracy ratio,  $\frac{I}{\mu}$ , and the profit margin when producing unadulterated products,  $r - c_n$ . Thus, we present results of Proposition 2 in Figure 4 in terms of these two factors. When the profit margin,  $r - c_n$ , is sufficiently high ( $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ , see part 2), it is optimal for the buyer to set its optimal contingent payment  $p^* = p^0 > c_n$ . To keep the supplier's adulteration probability  $x^*$  at bay, the buyer cannot fully extract the supplier's surplus. When the profit margin  $r - c_n$  is in the intermediate range ( $v_B - \sqrt{\frac{Iv_B}{\mu}} \geq r - c_n \geq \frac{v_B}{2} + \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ , see part 3) the buyer's optimal payment  $p^* = c_n$ , which is the minimum payment for the supplier to produce unadulterated products. In this case, the buyer fully extracts the supplier's surplus.



**Figure 4:** Optimal inspection contract as a function of inspection cost to inspection accuracy ratio,  $\frac{I}{\mu}$ , and profit margin,  $r - c_n$ .

We now examine the comparative statics of the optimal inspection contract  $(X^*, p^*)$ , the resulting equilibrium probabilities  $(x^*, y^*)$ , and profits with respect to the buyer's product liability  $v_B$ . While the comparative statics have been proven analytically, we shall omit formal proofs to simplify the exposition, and present results in Figure 5 instead. Under the optimal contract, the supplier's adulteration probability,  $x^*$ , is decreasing in  $v_B$  and that the buyer's inspection probability,  $y^*$ , is increasing in  $v_B$ . This result is due to the "mutual anticipation" of the supplier and the buyer in

equilibrium. That is, when the buyer's liability  $v_B$  becomes higher, the supplier anticipates that the buyer will inspect more; and hence, the supplier reduces its adulteration probability  $x^*$ . Interestingly, the optimal contingent payment  $p^*$  is decreasing in liability  $v_B$ . A possible explanation for this result is that instead of increasing the contingent payment  $p^*$  it is more effective to increase the inspection probability to provide the same incentive to the supplier not to adulterate the product. It is also interesting that the channel profit is not monotone in liability  $v_B$ . For small liability values, the supplier's profit is significant and for large liability values the buyer's profit is significant. But regardless of the value of  $v_B$ , for the same parameter values as were used in the discussion of the optimal deferred payment contract as in §4.3, the channel profit under the inspection contract is only a small fraction of the value of the centralized system's profit,  $r - c_n = 4$ . We shall discuss the choice between the deferred payment and inspection mechanism in Section 7.



**Figure 5:** Inspection optimal contract, comparative statics with respect to  $v_B$ .

**Legend:** Left panel: Optimal  $p^*$ , buyer's, supplier's, and channel profits. Right panel: Optimal  $x^*$  and  $y^*$ . Parameter values:  $r = 5$ ,  $c_n = 1$ ,  $c_d = 0.5$ ,  $I = 1$ , and  $\mu = 0.8$ . The centralized system profit is  $r - c_n = 4$ .

Because we are using the inspection mechanism primarily as the benchmark for the deferred payment and combined mechanisms, we omit discussion of other comparative statics. For example, consistent with the intuition, as the inspection cost,  $I$ , increases the equilibrium inspection probability increases, the equilibrium probability of producing adulterated products decreases, and the buyer's profit decreases.

## 6 Combined Inspection and Deferred Payment Contract

Recall from §3.3 that, under the combined mechanism, the buyer offers contract  $(X, p, q, T)$  to the supplier, where  $X \geq 0$  is the upfront payment. At time 0, if no adulteration is discovered (due to inspection or no inspection), then the buyer pays the first contingent payment,  $p$ . Otherwise, the buyer rejects the product and the game ends. If no adulteration is discovered at time 0, then the buyer presumes the product is unadulterated and sells the product at time 0, booking revenue  $r$ . Only if no adulteration is discovered by the customer by time  $T$ , the buyer pays the second contingent payment  $q$  at time  $T$ . If adulteration is discovered by the customer at any time, the buyer incurs the product liability, whose present value for the buyer is  $v_B$ .

By definition, the combined contract  $(X, p, q, T)$  has more decision variables than either the inspection or the deferred payment contracts. Hence, the buyer's profit under the optimal combined

contract should be greater than or equal to the maximum of the buyer's profits obtained in either the optimal inspection or the optimal deferred payment contracts. We establish the following interesting result in this section: the buyer's profit is exactly equal to the maximum of the buyer's profits obtained in either the optimal inspection or the optimal deferred payment contracts. Hence, the more complex combined contract is redundant, because one of the simpler contracts will suffice.

For any combined contract  $(X, p, q, T)$ , we now determine the equilibria (mixed and pure strategy). As in Section 5,  $x$  represents the supplier's adulteration probability ( $x \stackrel{\text{def}}{=} \Pr[a = d]$ ), and  $y$  represents the buyer's inspection probability ( $y \stackrel{\text{def}}{=} \Pr[i = 1]$ ). The profits of the supplier and the buyer for any combination of the pure strategies are given in Table 3. The following lemma presents the equilibria for any combined contract.

**Lemma 3.** *For any given combined contract  $(X, p, q, T) \geq 0$ , the equilibria of the combined mechanism can be described as follows.*

1. *If  $c_n - c_d < q_S(1 - \eta)$ , then the pure strategy  $(x^* = 0, y^* = 0)$  is a unique equilibrium. The corresponding equilibrium profits are*

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (17a)$$

$$\pi_B(X, p, q, T) = r - X - p - q_B. \quad (17b)$$

2. *If  $c_n - c_d = q_S(1 - \eta)$ , then any point  $(x, 0)$ , where  $x \in \left[0, \frac{I}{\mu(p + q_B\eta + v_B - r)}\right]$ , is an equilibrium. The supplier's and the buyer's equilibrium profits are*

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (18a)$$

$$\pi_B(X, p, q, T) = r - X - p - q_B - x[v_B - q_B(1 - \eta)]. \quad (18b)$$

3. *If  $q_S(1 - \eta) < c_n - c_d < q_S(1 - \eta) + \mu(p + q_S\eta)$ , then the combined inspection and deferred payment game has a unique mixed strategy equilibrium:  $x^* = \frac{I}{\mu(p + q_B\eta + v_B - r)}$  and  $y^* = \frac{c_n - c_d - q_S(1 - \eta)}{\mu(p + q_S\eta)}$ , where  $x^* \in (0, 1)$  and  $y^* \in (0, 1)$ . The supplier's and the buyer's equilibrium profits are*

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (19a)$$

$$\pi_B(X, p, q, T) = r - X - p - q_B - \frac{I[v_B - q_B(1 - \eta)]}{\mu(p + q_B\eta + v_B - r)}. \quad (19b)$$

4. *If  $c_n - c_d = q_S(1 - \eta) + \mu(p + q_S\eta)$ , then any point  $(x, 1)$ , where  $x \in \left[\frac{I}{\mu(p + q\eta + v_B - r)}, 1\right]$ , is an equilibrium of the combined inspection and deferred payment game. The corresponding supplier's and buyer's profits are*

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (20a)$$

$$\pi_B(X, p, q, T) = r - X - I - p - q_B - x[v_B - q_B(1 - \eta) - \mu(v_B + p + q_B\eta - r)]. \quad (20b)$$

5. If  $c_n - c_d > q_S(1 - \eta) + \mu(p + q_S\eta)$ , then the pure strategy  $(x^* = 1, y^* = 1)$  is a unique equilibrium. The corresponding equilibrium profits are

$$\pi_S(X, p, q, T) = X + (p + q_S\eta)(1 - \mu) - c_d, \quad (21a)$$

$$\pi_B(X, p, q, T) = -X - I - (v_B + p + q_B\eta - r)(1 - \mu). \quad (21b)$$

Because the combined mechanism combines the inspection and the deferred payment mechanisms, it is not surprising to see that the results stated in Lemma 3 resembled the results stated in Lemma 1 from the inspection mechanism. Therefore, Lemma 3 has a similar interpretation as Lemma 1. We omit the details.

Similar to part 3 of Lemma 1, part 2 and part 4 of Lemma 3 reveal that the combined mechanism has multiple equilibria when  $c_n - c_d = q_S(1 - \eta)$  or when  $c_n - c_d = q_S(1 - \eta) + \mu(p + q_S\eta)$ , respectively. By noting from part 2 that the buyer's profit is decreasing in the supplier's adulteration probability  $x$  because  $v_B \geq r \geq q_B > q_B(1 - \eta)$  and that the supplier's profit is independent of  $x$ , we can focus on a unique (payoff) dominant equilibrium by choosing the smallest  $x$  for the case as stated in part 2. Also, we can use the same approach to determine a unique (payoff) dominant equilibrium by choosing the smallest  $x$  for the case as stated in part 4. Consequently, we can rewrite Lemma 3 as the following corollary.

**Corollary 2.** *For any given combined contract  $(X, p, q, T) \geq 0$ , the payoff dominant equilibrium of the combined mechanism can be described as follows.*

1. If  $c_n - c_d \leq q_S(1 - \eta)$ , then the pure strategy  $(x^* = 0, y^* = 0)$  is a unique equilibrium. The corresponding equilibrium profits are

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (22a)$$

$$\pi_B(X, p, q, T) = r - X - p - q_B. \quad (22b)$$

2. If  $q_S(1 - \eta) < c_n - c_d \leq q_S(1 - \eta) + \mu(p + q_S\eta)$ , then the combined inspection and deferred payment game has a unique mixed strategy equilibrium:  $x^* = \frac{I}{\mu(p + q_B\eta + v_B - r)}$  and  $y^* = \frac{c_n - c_d - q_S(1 - \eta)}{\mu(p + q_S\eta)}$ , where  $x^* \in (0, 1)$  and  $y^* \in (0, 1)$ . The supplier's and the buyer's profits are

$$\pi_S(X, p, q, T) = X + p + q_S - c_n, \quad (23a)$$

$$\pi_B(X, p, q, T) = r - X - p - q_B - \frac{I[v_B - q_B(1 - \eta)]}{\mu(p + q_B\eta + v_B - r)}. \quad (23b)$$

3. If  $c_n - c_d > q_S(1 - \eta) + \mu(p + q_S\eta)$ , then the pure strategy  $(x^* = 1, y^* = 1)$  is a unique equilibrium. The corresponding equilibrium profits are

$$\pi_S(X, p, q, T) = X + (p + q_S\eta)(1 - \mu) - c_d, \quad (24a)$$

$$\pi_B(X, p, q, T) = -X - I - (v_B + p + q_B\eta - r)(1 - \mu). \quad (24b)$$

Corollary 2 resembles Corollary 1 under the inspection mechanism. For instance, when the cost gap ( $c_n - c_d$ ) is as in part 2, the supplier has incentive to cheat. In this case, only the buyer's optimal inspection probability  $y^*$  depends on cost gap ( $c_n - c_d$ ) in equilibrium as observed in parts 2 and 3 of Proposition 2 under the inspection mechanism.

Corollary 2 states the buyer's and the supplier's payoffs for any given combined contract  $(X, p, q, T)$ . We now determine the optimal combined contract by using the same approach as in Section 4 to determine the optimal deferred payment contract by solving the buyer's problem. Unlike problem (3), which has 3 decision variables, the buyer's problem associated with the combined contract has 4 decision variables:  $(X, p, q, T)$ . Nevertheless, we manage to solve the buyer's problem analytically and determine the optimal combined contract in the following Proposition.

**Proposition 3.** *The optimal combined contract  $(X^*, p^*, q^*, T^*)$  can be described as follows. Either  $(X^*, p^*, q^*, T^*) = (0, p^*, 0, 0)$  or  $(X^*, p^*, q^*, T^*) = (0, 0, q^*, T^*)$ , where  $p^*$  is the optimal contingent payment under the inspection mechanism as given in Proposition 2 and  $(q^*, T^*)$  are the optimal deferred contingent payment and deferred payment duration under the optimal deferred payment contract as given in Proposition 1.*

Even though the combined contract has more decision variables than the inspection contract and the deferred payment contract as discussed in earlier sections, Proposition 3 reveals that we can retrieve the optimal combined contract from the optimal inspection contract or the optimal deferred payment contract established earlier. The buyer's profit under the optimal combined contract is equal to the profit that the buyer can obtain from choosing the "dominant mechanism" between either the pure inspection mechanism or the pure deferred payment mechanism as discussed in Section 7. Hence, it is unnecessary for the buyer to offer the combined mechanism. From the perspective of mathematical programming, this result is unexpected because more decision variables should enable the buyer to obtain a higher profit. However, from the perspective of game theory, this result can be due to the fact that the supplier can choose between only two actions: adulterate or not. Because there are only two discrete actions that the supplier can choose from, having more decision variables by combining inspection and deferred payment may not help the buyer to obtain a higher profit (Png, 2010).

## 7 Choosing the Mechanism

Now that we know that the combined mechanism is redundant in our setting and recognizing that the inspection mechanism is commonly used in practice, we would like to examine conditions under which the deferred payment mechanism dominates the inspection mechanism.

Proposition 1 presents the optimal contract and the corresponding profits for the deferred payment mechanism. Proposition 2 does that for the inspection mechanism. To keep discussion interesting, we shall focus on the cases when the buyer's profit is positive under at least one of

these mechanisms.

From Propositions 1 and 2, parameters  $a = \frac{\alpha_S - \alpha_B}{\lambda}$  and  $c_d$  affect only the deferred payment mechanism, and parameters  $\frac{I}{\mu}$  and  $v_B$  affect only the inspection mechanism. This observation facilitates the following analysis.

**Effect of the inspection cost and the inspection accuracy.** Based on the intuitive findings in §5, the buyer's profit under the inspection mechanism is decreasing in ratio of the inspection cost over inspection accuracy,  $\frac{I}{\mu}$ , and the buyer's profit under the deferred payment mechanism is unaffected. Ultimately, if  $\frac{I}{\mu} > \frac{v_B}{4}$ , the buyer's profit under the inspection mechanism becomes zero. Therefore, an intuitive conclusion is that *high inspection cost or low inspection accuracy encourages the use of the deferred payment mechanism.*

**Effect of the buyer's product liability.** Again, using comparative statics results from §5, the buyer's profit under the inspection mechanism is increasing in liability  $v_B$  and the buyer's profit under the deferred payment mechanism is unaffected. If the liability is too small (for example,  $\frac{v_B}{4} < \frac{I}{\mu}$ ), the buyer's profit under the inspection mechanism is zero. Thus, a less intuitive takeaway from our analysis is that *for low-liability businesses, deferred payment is a better mechanism.*

**Effect of the financing cost gap and the defect discovery rate.** Next, let's turn to the parameters that affect the buyer's profit under the deferred payment mechanism only. From the analysis in §4, the buyer's profit under the deferred payment mechanism is decreasing in the financing cost gap  $\alpha_S - \alpha_B$  and is increasing in the rate of adulteration discovery by customers,  $\lambda$  (these parameters appear together as  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ ). If  $a = \frac{\alpha_S - \alpha_B}{\lambda}$  is very large, then the buyer makes zero profit under the deferred payment mechanism. Therefore, two very intuitive conclusions from our model are *for systems with large financing cost gap and slow rate of defect discovery by customers, inspection is a better mechanism.*

**Effect of the defective product cost.** Under the deferred payment mechanism the buyer's profit is increasing in the cost of the defective product,  $c_d$ , while the profit under the inspection mechanism is unaffected. Thus, as the cost of defective product  $c_d$  is getting closer to the cost of non-defective product  $c_n$ , the deferred payment becomes a more attractive mechanism for the buyer. This does not necessarily mean that the buyer would choose the deferred payment mechanism over the inspection mechanism. In this case, the choice depends on the value of other problem parameters and one needs to compare corresponding profit expressions to determine the best mechanism. The following propositions present sufficient conditions for the buyer to prefer the deferred payment mechanism.

**Proposition 4.** Consider the case when  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ .

Suppose  $\frac{c_d}{c_n} > \frac{a}{1+a}$ . Then the deferred payment mechanism dominates the inspection mechanism if  $\frac{I}{\mu} \geq \frac{c_n}{2} \left[ \left( \frac{c_d}{c_n} \right)^{-a} - 1 \right]$ .

Suppose  $\frac{c_d}{c_n} < \frac{a}{1+a}$ . Then the deferred payment mechanism dominates the inspection mechanism if  $\frac{I}{\mu} \geq \frac{c_n}{2} \left[ \left(1 - \frac{c_d}{c_n}\right) (1+a) \left(\frac{a}{1+a}\right)^{-a} - 1 \right]$ .

Although we cannot make as an unequivocal a statement as we did earlier, the overall direction of the result is that *the buyer is more likely to rely on the deferred payment mechanism if the cost of adulterated product is close to the cost of unadulterated product.*

Finally, let's study informational aspects of the mechanism choice. Specifically, to implement contracts in practice we need to consider which information is available to the buyer (even though the model does not directly address asymmetric information other than actions of the supplier). Likely, the values for revenues  $r$ , liabilities  $v_B$ , inspection cost  $I$ , inspection accuracy  $\mu$ , financing gap  $\alpha_S - \alpha_B$  are known to the buyer. The buyer may also know the supplier's cost of producing non-defective products  $c_n$ . However, the buyer is unlikely to know the cost of producing defective products  $c_d$ . Thus, contracts that do not require this knowledge are preferable for the buyer. By this criterion, the buyer prefers the inspection mechanism to the deferred payment mechanism because the optimal contract terms in the inspection contract do not depend on  $c_d$ . However, the deferred payment mechanism might provide a good solution in the region corresponding to part 1 of Proposition 1, where  $q_S^* = c_n$ . If  $T^*$  is set as an industry standard and as long as the buyer is confident that the cost ratio  $\frac{c_d}{c_n}$  is within certain bounds, (as we discussed in §4.3) this mechanism is quite robust in deterring supplier's adulteration. Furthermore, while the inspection contract does not require knowledge of  $c_d$ , the inspection probability depends on this value. Therefore, while the buyer can offer the optimal inspection contract without the knowledge of  $c_d$ , the buyer cannot implement this contract.

Alternatively, if the buyer does not know how defective product will affect the customers (i.e., the buyer does not know  $v_B$ ), then the deferred payment mechanism might be preferable because it eliminates the incentive for the supplier to produce adulterated product entirely.

## 8 Conclusions

Recognizing the fact that pre-production process certification is insufficient to deter suppliers from product adulteration and that foreign supplier's liability and warranty are difficult to enforce or implement, we have studied three forms of contingent payment mechanisms that are intended to deter product adulteration: the deferred payment, the inspection, and the combined mechanisms. To examine the extent to which these mechanisms deter suppliers from adulteration, we have analyzed inspection sub-games for the inspection and combined mechanisms, derived analytical expressions for the optimal contracts in all mechanisms, and studied effects of various parameters on the performance of these mechanisms.

For the optimal deferred payment mechanism, we observed that as the cost of producing adulterated products decreases so that the incentive to cheat for the supplier increases, the buyer manages

that incentive first by extending the deferred payment duration and then by raising the contingent payment amount. The deferred payment mechanism can be implemented via a common financial contract — trade credit. We showed that under reasonable assumptions on the financing cost gap, the unadulterated/adulterated product cost gap, and the rate at which customers discover adulteration, a “net 30” trade credit contract (a popular example of trade credit contracts) can be effective at controlling the supplier’s incentives. Some analytical predictions of our model are consistent with the prior empirical studies on the use of trade credit contracts for product quality control. In particular, the lower the rate of defects discovery the longer the trade credit durations observed empirically.

We use the inspection mechanism as the benchmark for the performance of the deferred payment mechanism. Still, several interesting observations can be made about the inspection mechanism itself. For instance, we have shown that the optimal contingent payment under the inspection mechanism decreases in the buyer’s liability (Intuitively, when the buyer’s liability increases one would expect that the buyer would increase its contingent payment to entice the supplier not to adulterate the product). The explanation for this result is that the equilibrium inspection probability under the optimal contract is increasing in the buyer’s liability, so the contingent payment itself can be lower. Under the optimal inspection mechanism, we have found that there is always a positive probability of the supplier producing adulterated product, which ultimately ends up with the customer. In contrast, we have shown that the optimal deferred payment mechanism can deter the product adulteration entirely.

Intuitively, one would expect that a more complex mechanism that combines the deferred payment and inspection mechanisms would generate higher profits for the buyer, who can select the best combination of both mechanisms. Interestingly, while the combined contract has more decision variables, we have shown that the buyer’s profit under the optimal combined contract is equal to the profit that the buyer can obtain from choosing the “dominant mechanism” between either the pure inspection mechanism or the pure deferred payment mechanism. Hence, it is unnecessary for the buyer to offer the combined mechanism.

By comparing the buyer’s profits obtained under the deferred payment and inspection mechanisms, we have established conditions under which one mechanism dominates the other. Specifically, the dominance of one mechanism over the other mechanism is based on four key factors: (a) inspection cost relative to inspection accuracy; (b) buyer’s liability (c) difference in financing rates relative to the rate at which customers discover defects; and (d) incentive for the supplier to cheat that is measured in terms of cost gap between unadulterated and adulterated products. The influence of the factors (a) and (c) is intuitive. High inspection cost or low inspection accuracy make the deferred payment mechanism more attractive. High financing gap or low defects discover rate

make the inspection mechanism more attractive. The role of factors (b) and (d) is more interesting. Either low buyer's liability for defective products or low cost gap, both meaning that the threat from adulteration is low, make the deferred payment mechanism preferable for the buyer over the inspection mechanism.

In future research, one could explore a number of directions. In this paper, we have developed a model of a situation in which a supplier has to choose between producing adulterated products or not. Hence, the supplier has only two potential actions to choose from. In other settings, suppliers may have a continuum of choices. We do not think that the main findings of this paper would change, but the analysis would have to be modified. In our model, we have assumed that the inspection effectiveness is exogenous; however, it is of interest to consider the case in which the buyer can improve inspection accuracy with additional investments as examined in Baiman et al. (2000). Next, in our model, we have assumed that for practical reasons, the buyer is responsible for the entire product liability should the product turns out to be adulterated. In view of the recent episodes of product adulteration and the fact that the manufacturer can rarely claim supplier's product liability, this assumption captures the reality well. However, it is of interest to examine a situation in which the supplier shares the product liability with the buyer, as examined in Chao et al. (2009). Supplier liability may reflect the loss of future business with the buyer, if the supplier is caught adulterating products. We believe that incorporating supplier liability into the model will not change our main conclusions. Next, to capture the reality of the opportunistic behavior of the overseas suppliers, we used a static contracting model (in this regard, our model is similar to the majority of the contracting literature). Both suppliers and the buyers in practice behave as if they had very short-term objectives (hence, the record number of product recalls due to adulteration) and frequently, buyers do not have much history with the suppliers to that can be useful to form deterrent strategies. Therefore, a static contracting model is appropriate. However, one can also think of situations where the buyer and the supplier interact repeatedly over a very long time. Analytically, this creates difficulties because of the multiplicity of equilibria due to the Folk Theorem. Also, the theory of dynamic games with asymmetric information is not as well developed as the theory of static games. Clearly, there are many other interesting extensions (e.g., asymmetric information about the supplier's cost or production capability, asymmetric information about the buyer's demand, the comparison of the deferred payment mechanism with product warranty mechanism). These are left for future research.

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# Appendix for “Managing Opportunistic Supplier Product Adulteration: Deferred Payments, Inspection, and Combined Mechanisms”

## A Proofs

### *Proof of Proposition 1:*

In (4), for given  $Y$  and  $T$ , one would like to reduce  $q_S$ . Two constraints prevent us from setting  $q_S = 0$ . Which of the two constraints is binding depends on the relative values of  $\frac{c_n - c_d}{1 - e^{-\lambda T}}$  and  $c_n - Y$ .

**Case 1:** For this case,  $\frac{c_n - c_d}{1 - e^{-\lambda T}} \geq c_n - Y$ . Then  $q_S^* = \frac{c_n - c_d}{1 - e^{-\lambda T}}$  and the buyer’s problem becomes

$$\max_{Y, T} \quad r - Y - \frac{c_n - c_d}{1 - e^{-\lambda T}} e^{(\alpha_S - \alpha_B)T} \quad (\text{A-1a})$$

$$\text{s.t.} \quad Y \geq c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}}, \quad Y \geq 0, \quad T \geq 0. \quad (\text{A-1b})$$

One would like to reduce  $Y$ . There are two subcases:  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} > 0$  and  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \leq 0$ .

**Case 1a:** For this subcase,  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} > 0$  (equivalently,  $e^{-\lambda T} < \frac{c_d}{c_n}$ ). Therefore,  $Y^* = c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}}$  and the buyer’s problem becomes

$$\max_T \quad r - c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \left[ e^{(\alpha_S - \alpha_B)T} - 1 \right] \quad (\text{A-2a})$$

$$\text{s.t.} \quad e^{-\lambda T} < \frac{c_d}{c_n}, \quad T \geq 0. \quad (\text{A-2b})$$

To solve problem (A-2), define  $u \stackrel{\text{def}}{=} e^{-\lambda T}$ , and replace variable  $T$  with variable  $u$ . The resulting optimization problem is

$$\max_u \quad r - c_n - \frac{c_n - c_d}{1 - u} \left[ u^{-a} - 1 \right] \quad (\text{A-3a})$$

$$\text{s.t.} \quad u < \frac{c_d}{c_n}, \quad (\text{A-3b})$$

where  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . Let  $f(u) = (1 - u^{-a})(1 - u)^{-1}$ . Compute,  $f'(u) = \frac{u^{-(a+1)}}{(1-u)^2} [u^{a+1} - u + a(1 - u)]$ . Because the term  $[u^{a+1} - u + a(1 - u)]$  is strictly positive for any  $u \in (0, 1)$ , function  $f(u)$  is strictly increasing over  $u \in (0, 1)$ . Combining this observation with the fact that  $\frac{c_d}{c_n} < 1$ , we conclude that the optimal solution to problem (A-3) is  $u^* = \frac{c_d}{c_n}$ .

To summarize the solution for subcase 1a:  $e^{-\lambda T^*} = \frac{c_d}{c_n}$ ,  $Y^* = 0$ , and  $q_S^* = c_n$ .

**Case 1b:** For this subcase,  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \leq 0$  (equivalently,  $e^{-\lambda T} \geq \frac{c_d}{c_n}$ ). Therefore,  $Y^* = 0$  and the buyer’s problem becomes

$$\max_T \quad r - \frac{c_n - c_d}{1 - e^{-\lambda T}} e^{(\alpha_S - \alpha_B)T} \quad (\text{A-4a})$$

$$\text{s.t.} \quad e^{-\lambda T} \geq \frac{c_d}{c_n}, \quad T \geq 0. \quad (\text{A-4b})$$

As we did for case 1a, replace variable  $T$  with  $u = e^{-\lambda T}$  to obtain

$$\max_u \quad r - \frac{c_n - c_d}{1 - u} u^{-a} \quad (\text{A-5a})$$

$$\text{s.t.} \quad u \geq \frac{c_d}{c_n}, \quad u \leq 1. \quad (\text{A-5b})$$

where  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . The first derivative of the objective value function  $f(u) = r - \frac{c_n - c_d}{1 - u} u^{-a}$  is

$$f'(u) = \frac{(c_n - c_d)u^{-(a+1)}[a(1 - u) - u]}{(u - 1)^2} \quad (\text{A-6})$$

This derivative is  $f'(u) > 0$  for  $u < \frac{a}{1+a}$  and is  $f'(u) \leq 0$  for  $u \geq \frac{a}{1+a}$ . Observe  $\frac{a}{1+a} < 1$ . Therefore, if  $\frac{c_d}{c_n} > \frac{a}{1+a} = \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$  then  $f(u)$  is decreasing everywhere on  $u \in \left[\frac{c_d}{c_n}, 1\right]$ . If  $\frac{c_d}{c_n} < \frac{a}{1+a}$ , then function  $f(u)$  achieves maximum at  $u = \frac{a}{1+a} \in \left[\frac{c_d}{c_n}, 1\right]$ .

Summary for subcase 1b: If  $\frac{c_d}{c_n} \geq \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$ , then for this subcase,  $e^{-\lambda T^*} = \frac{c_d}{c_n}$ ,  $Y^* = 0$ , and  $q_S^* = c_n$ . If  $\frac{c_d}{c_n} < \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$ , then for this subcase,  $e^{-\lambda T^*} = \frac{\alpha_S - \alpha_B}{\lambda + \alpha_S - \alpha_B}$ ,  $Y^* = 0$ , and  $q_S^* = \frac{(c_n - c_d)(\lambda + \alpha_S - \alpha_B)}{\lambda}$ .

**Case 2:** For this case,  $\frac{c_n - c_d}{1 - e^{-\lambda T}} < c_n - Y$ . Then,  $q_S^* = c_n - Y$  and the buyer's problem becomes

$$\max_{Y, q_S, T} \quad r - Y - (c_n - Y)e^{(\alpha_S - \alpha_B)T} = r + Y \left[ e^{(\alpha_S - \alpha_B)T} - 1 \right] - c_n e^{(\alpha_S - \alpha_B)T} \quad (\text{A-7a})$$

$$\text{s.t.} \quad Y < c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}}, \quad Y \geq 0, \quad T \geq 0. \quad (\text{A-7b})$$

For this problem, if  $T$  is fixed, we would like to increase  $Y$ . There are two subcases to consider:

$c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \geq 0$  and  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} < 0$ .

**Case 2a:** For this subcase  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \geq 0$  (equivalently  $e^{-\lambda T} \leq \frac{c_d}{c_n}$ ). Therefore,  $Y^* = c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}}$  and the buyer's problem becomes

$$\max_T \quad r - c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} \left[ e^{(\alpha_S - \alpha_B)T} - 1 \right] \quad (\text{A-8a})$$

$$\text{s.t.} \quad e^{-\lambda T} \leq \frac{c_d}{c_n}, \quad T \geq 0. \quad (\text{A-8b})$$

This is the subcase 1a that we already solved.

**Case 2b:** For this subcase  $c_n - \frac{c_n - c_d}{1 - e^{-\lambda T}} < 0$  and there is no feasible solution.

Combining results from cases 1a, 1b, 2a, and 2b, we derive the statement of the proposition.  $\blacksquare$

### **Proof of Lemma 1:**

Omitting explicit dependence on  $(X, p)$ , the supplier's expected profit  $\pi_S^{(x,y)}$ , if the players choose probabilities  $(x, y)$  for their pure strategies (see Table 2), can be expressed as:

$$\begin{aligned} \pi_S^{(x,y)} &= X + x(1 - y)(p - c_d) + (1 - x)(1 - y)(p - c_n) \\ &\quad + xy[p(1 - \mu) - c_d] + (1 - x)y(p - c_n) = X + p - c_n + x(c_n - c_d - y\mu p). \end{aligned} \quad (\text{A-9})$$

Similarly, the buyer's expected profit  $\pi_B^{(x,y)}$  can be expressed as:

$$\begin{aligned} \pi_B^{(x,y)} = & -X + r - x(1-y)(p+v_B) - (1-x)(1-y)p - xy[I + p(1-\mu) - r\mu \\ & + v_B(1-\mu)] - (1-x)y(I+p) = -X + r - p - xv_B - y[I - x\mu(p+v_B-r)]. \end{aligned} \quad (\text{A-10})$$

If  $p < \frac{c_n - c_d}{\mu}$ , then, regardless of the value of  $y \in [0, 1]$ , the supplier's optimal choice is to produce defective products:  $x^* = 1$ . By Assumption 3 and the fact that  $p \geq 0$ , inequality  $p + v_B - r > I/\mu$  is true. Therefore, when  $x = 1$ , it is optimal for the buyer to inspect:  $y^* = 1$ . Therefore, there is and unique equilibrium in pure strategies, where the supplier produces a defective product and the buyer inspects:  $(a = d, i = 1)$ .

If  $p \geq \frac{c_n - c_d}{\mu}$ , when  $c_n - c_d - yp\mu = 0$  or equivalently  $y = \frac{c_n - c_d}{p\mu} \in (0, 1]$ , the supplier can choose any value for  $x \in [0, 1]$ . Similarly, when  $I - x\mu(p + v_B - r) = 0$  or equivalently  $x = \frac{I}{\mu(p + v_B - r)}$ , the buyer can choose any value for  $y \in [0, 1]$ .

If  $p > \frac{c_n - c_d}{\mu}$ , then the best response curves for the buyer and the supplier intersect at  $x^* = \frac{I}{\mu(p - r + v_B)}$ ,  $y^* = \frac{c_n - c_d}{p\mu}$  only. If  $p = \frac{c_n - c_d}{\mu}$ , then the best response curves for the buyer and the supplier overlap for all  $(x, 1)$ , where  $x \in \left[ \frac{I}{\mu(p - r + v_B)}, 1 \right]$ .

The equilibrium profits are computed by substituting equilibrium  $(x^*, y^*)$  into (A-9) and (A-10). ■

### ***Proof of Lemma 2:***

First, let us examine region 1 in Figure 3. When  $0 \leq p < \frac{c_n - c_d}{\mu}$ , we use (7) for the buyer's and the supplier's profits and the buyer's problem is given as:

$$\max \quad \pi_B(X, p) = -X - I + (r - p - v_B)(1 - \mu) \quad (\text{A-11a})$$

$$\text{s.t.} \quad \pi_S(X, p) = X + p(1 - \mu) - c_d \geq 0, \quad (\text{A-11b})$$

$$X \geq 0, \text{ and } \frac{c_n - c_d}{\mu} > p \geq 0. \quad (\text{A-11c})$$

For any given  $p$ , the constraint  $X + p(1 - \mu) - c_d \geq 0$  is binding so that the optimal  $X$  must satisfies  $X = c_d - p(1 - \mu)$ . By substituting  $X = c_d - p(1 - \mu)$  into the objective function, the objective function becomes a constant, namely,  $\pi_B(X, p) = -I - c_d - (v_B - r)(1 - \mu) < 0$ .

Second, let us examine region 2. When  $c_n \geq p \geq \frac{c_n - c_d}{\mu}$ , we use (8) for the buyer's and supplier's profits and the buyer's problem can be rewritten as:

$$\max \quad \pi_B(X, p) = -X + r - p - \frac{Iv_B}{(p - r + v_B)\mu} \quad (\text{A-12a})$$

$$\text{s.t.} \quad \pi_S(X, p) = X + p - c_n \geq 0, \quad (\text{A-12b})$$

$$X \geq 0, \text{ and } c_n \geq p \geq \frac{c_n - c_d}{\mu}. \quad (\text{A-12c})$$

For any given  $p$ , the supplier's individual rationality constraint is binding so that  $X + p = c_n$ . By considering this binding constraint, the buyer's problem becomes

$$\max \quad r - c_n - \frac{Iv_B}{(p - r + v_B)\mu} \quad (\text{A-13a})$$

$$\text{s.t. } p \in \left[ \frac{c_n - c_d}{\mu}, c_n \right]. \quad (\text{A-13b})$$

By noting that the objective function is increasing in  $p$ , it is optimal to set  $p = c_n$ . Therefore, the optimal contract in region 2 is  $(X, p) = (0, c_n)$ , and the buyer's optimal profit is

$$\pi_B(0, c_n) = r - c_n - \frac{Iv_B}{(c_n - r + v_B)\mu}. \quad (\text{A-14})$$

It remains to examine region 3. We augment region 3 by including the case when  $p = c_n$  so that region 3 contains the optimal solution of region 2. In this case, we can use the same approach for the analysis of region 2 to show that the constraint  $X \geq 0$  is binding and the buyer's problem can be simplified as:

$$\max \quad r - p - \frac{Iv_B}{(p - r + v_B)\mu} \quad (\text{A-15a})$$

$$\text{s.t. } p \geq c_n. \quad (\text{A-15b})$$

By noting that the objective function is concave in  $p$ . We can use the first-order condition to show that the optimal  $p$  in the augmented region 3 (which includes the optimal solution of region 2) is given by  $X = 0$ , and  $p = \max\{c_n, p^0\}$ , where  $p^0 \stackrel{\text{def}}{=} r - v_B + \sqrt{\frac{Iv_B}{\mu}}$ . By substituting the optimal solution to the objective function and the supplier's payoff, we complete our proof.  $\blacksquare$

### ***Proof of Proposition 2:***

Recall from (11) that the buyer's optimal profit in region 1 is  $\pi_B^{(1)}(X^{(1)}, p^{(1)}) < 0$ . It suffices to examine the optimal inspection contract  $(X^{(2)}, p^{(2)}) = (0, p^{(2)})$  and the buyer's optimal profit  $\pi_B^{(2)}(X^{(2)}, p^{(2)})$  in regions 2 and 3 as stated in statement (2) of Lemma 2.

In preparation, observe from statement (2) of Lemma 2 that the optimal contingent payment  $p^{(2)}$  satisfies

$$p^{(2)} = \begin{cases} p^0 = r - v_B + \sqrt{\frac{Iv_B}{\mu}} & \text{if } r - c_n \geq v_B - \sqrt{\frac{Iv_B}{\mu}} \\ c_n & \text{if } r - c_n < v_B - \sqrt{\frac{Iv_B}{\mu}}, \end{cases} \quad (\text{A-16})$$

and that the buyer's optimal profit in regions 2 and 3 can be rewritten as:

$$\pi_B^{(2)}(X^{(2)}, p^{(2)}) = \begin{cases} v_B - 2\sqrt{\frac{Iv_B}{\mu}} & \text{if } r - c_n \geq v_B - \sqrt{\frac{Iv_B}{\mu}} \\ r - c_n - \frac{Iv_B}{(c_n + v_B - r)\mu} & \text{if } r - c_n < v_B - \sqrt{\frac{Iv_B}{\mu}} \end{cases} \quad (\text{A-17})$$

In this case,  $v_B - 2\sqrt{\frac{Iv_B}{\mu}} > 0$  if and only if  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n - \frac{Iv_B}{(c_n + v_B - r)\mu} > 0$  if and only if  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n \in \left[ \frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}, \frac{v_B}{2} + \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}} \right]$ .

Before proceeding with the proof of the proposition, it is useful to establish relative values of  $\frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ ,  $\frac{v_B}{2} + \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$  and  $v_B - \sqrt{\frac{Iv_B}{\mu}}$ , which we do in the following lemma.

**Lemma 4.** *The following are true:  $\frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}} \leq v_B - \sqrt{\frac{Iv_B}{\mu}} \leq \frac{v_B}{2} + \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ .*

*Proof.* We can rewrite the statement of the lemma as  $-\sqrt{\frac{v_B^2}{4} - \frac{Iv_B}{\mu}} \leq \frac{v_B}{2} - \sqrt{\frac{Iv_B}{\mu}} \leq \sqrt{\frac{v_B^2}{4} - \frac{Iv_B}{\mu}}$ . This is equivalent to  $-\sqrt{x-y} \leq \sqrt{x} - \sqrt{y} \leq \sqrt{x-y}$ . To prove  $\sqrt{x} - \sqrt{y} \leq \sqrt{x-y}$ , move  $\sqrt{y}$  to the RHS and square both sides, obtaining  $x \leq x-y+y+2\sqrt{y(x-y)}$ , which is true. To prove  $-\sqrt{x-y} \leq \sqrt{x} - \sqrt{y}$ , rewrite it as  $\sqrt{y} \leq \sqrt{x} + \sqrt{x-y}$  and square, obtaining  $y \leq x+x-y+2\sqrt{x(x-y)}$ . This is equivalent to  $0 \leq x-y+\sqrt{x(x-y)}$ . Because  $x-y \geq 0$ , the latter is true. ■

We now prove the proposition by examining the buyer's profit function  $\pi_B^{(2)}(X^{(2)}, p^{(2)})$  given in (A-17). First, when  $\frac{I}{\mu} > \frac{v_B}{4}$ , from above  $\pi_B^{(2)}(X^{(2)}, p^{(2)}) < 0$ . Combining this observation with the fact that  $\pi_B^{(1)}(X^{(1)}, p^{(1)}) < 0$ , we conclude that the buyer's optimal profit is always negative. Hence, the buyer should not participate.

Second, consider the case when  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ . In this case, from (A-16) it follows that  $p^{(2)} = p^0$ , and from (A-17) and above it follows that  $\pi_B^{(2)}(0, p^0) = v_B - 2\sqrt{\frac{Iv_B}{\mu}} > 0$ . Therefore,  $(X^*, p^*) = (0, p^0)$  is the optimal inspection contract, and the buyers' and the supplier's profits are as stated in statement 2.

Third, when  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n$  satisfies  $v_B - \sqrt{\frac{Iv_B}{\mu}} \geq r - c_n \geq \frac{v_B}{2} + \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ , from (A-16) it follows that  $p^{(2)} = c_n$ , and from (A-17) and above it follows that  $\pi_B^{(2)}(0, c_n) = r - c_n - \frac{Iv_B}{(c_n + v_B - r)\mu} > 0$ . Therefore,  $(X^*, p^*) = (0, c_n)$  is the optimal inspection contract, and the buyers' and the supplier's profits are as stated in statement 3.

Finally, when  $\frac{I}{\mu} \leq \frac{v_B}{4}$  and  $r - c_n < \frac{v_B}{2} - \frac{1}{2}\sqrt{v_B^2 - 4\frac{Iv_B}{\mu}}$ , from (A-16) it follows that  $p^{(2)} = c_n$ , and from (A-17) and above it follows that  $\pi_B^{(2)}(0, c_n) = r - c_n - \frac{Iv_B}{(c_n + v_B - r)\mu} < 0$ . Combining this observation with the fact that  $\pi_B^{(1)}(X^{(1)}, p^{(1)}) < 0$ , we conclude that the buyer's optimal profit is always negative. Hence, the buyer should not participate. This completes the proof. ■

### ***Proof of Lemma 3:***

Omitting explicit dependence on  $(X, p, q, T)$ , the supplier's expected profit  $\pi_S^{(x,y)}$  if the players choose probabilities  $(x, y)$  for their pure strategies (see Table 3), can be written as:

$$\pi_S^{(x,y)} = X + p + q_S - c_n + x[c_n - c_d - q_S(1 - \eta) - y\mu(p + q_S\eta)]. \quad (\text{A-18})$$

Similarly, the buyer's expected profit  $\pi_B^{(x,y)}$  can be written as:

$$\pi_B^{(x,y)} = r - X - p - q_B - x[v_B - q_B(1 - \eta)] - y[I - x\mu(p + q_B\eta) + v_B - r]. \quad (\text{A-19})$$

If  $c_n - c_d < q_S(1 - \eta)$ , then, because  $p$  and  $q$  are non-negative, regardless of the value of  $y \in [0, 1]$ , the supplier's optimal choice is to produce non-defective products:  $x^* = 0$ . The supplier's best response is not to inspect:  $y^* = 0$ . Therefore, there is a unique equilibrium in pure strategies, where the supplier produces a non-defective product and the buyer does not inspect:  $(a = n, i = 0)$ .

If  $c_n - c_d = q_S(1 - \eta)$ , then for  $y > 0$  the supplier's optimal choice is to produce non-defective products:  $x^* = 0$ . For  $y = 0$ , the supplier can choose any value  $x^* \in [0, 1]$ . The buyer's best response is not to inspect:  $y^* = 0$ , when  $x < \frac{I}{\mu(p+q_B\eta+v_B-r)}$ ; inspect:  $y^* = 1$ , when  $x > \frac{I}{\mu(p+q_B\eta+v_B-r)}$ ; and  $y^* \in [0, 1]$ , when  $x = \frac{I}{\mu(p+q_B\eta+v_B-r)}$ . Thus, there are multiple equilibria:  $(x, 0)$ , where  $x \in \left[0, \frac{I}{\mu(p+q_B\eta+v_B-r)}\right]$ .

If  $c_n - c_d > q_S(1 - \eta)$ , then we need to consider subcases  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) > 0$ ,  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) = 0$ , and  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) < 0$ .

If  $c_n - c_d > q_S(1 - \eta)$  and  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) > 0$ , then, regardless of the value of  $y \in [0, 1]$ , the supplier's optimal choice is to produce defective products:  $x^* = 1$ . But when  $x = 1$ , it is optimal for the buyer to inspect:  $y^* = 1$ . Therefore, there is a unique equilibrium in pure strategies, where the supplier produces a defective product and the buyer inspects:  $(a = d, i = 1)$ .

If  $c_n - c_d > q_S(1 - \eta)$  and  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) < 0$ , the best response curves for the buyer and the supplier intersect once at  $x^* = \frac{I}{\mu(p+q_B\eta+v_B-r)}$  and  $y^* = \frac{c_n - c_d - q_S(1 - \eta)}{\mu(p + q_S\eta)}$ .

If  $c_n - c_d > q_S(1 - \eta)$  and  $c_n - c_d - q_S(1 - \eta) - \mu(p + q_S\eta) < 0$ , then the best response curves for the buyer and the supplier overlap for all  $(x, 1)$ , where  $x \in \left[\frac{I}{\mu(p+q_B\eta+v_B-r)}, 1\right]$ .

The equilibrium profits are computed by substituting equilibrium  $(x^*, y^*)$  into (A-18) and (A-19). ■

### ***Proof of Proposition 3:***

Consider the three parts in Corollary 2. In part 3, when  $c_n - c_d > q_S(1 - \eta) + \mu(p + q_S\eta)$ , the buyer's profit is negative, so this case need not be considered.

In part 1, when  $c_n - c_d \leq q_S(1 - \eta)$  the buyer's optimal contract design problem is the same as (4) with  $Y = X + p$ . Therefore, the optimal contract for this case is given in Proposition 1.

Finally consider part 2. If  $q_S(1 - \eta) < c_n - c_d \leq q_S(1 - \eta) + \mu(p + q_S\eta)$ , then the buyer's optimal contract problem is

$$\max_{X,p,q,T} \quad r - X - p - q_B - \frac{I[v_B - q_B(1 - \eta)]}{\mu(p + q_B\eta + v_B - r)} \quad (\text{A-20a})$$

$$\text{s.t.} \quad X + p + q_S - c_n \geq 0, \quad (\text{I.R. constraint}) \quad (\text{A-20b})$$

$$q_S(1 - \eta) < c_n - c_d, \quad (\text{A-20c})$$

$$q_S(1 - \eta) + \mu(p + q_S\eta) \geq c_n - c_d, \quad (\text{A-20d})$$

$$X \geq 0, p \geq 0, q \geq 0, T \geq 0. \quad (\text{A-20e})$$

Recall that  $\eta = e^{-\lambda T}$ ,  $q_B = qe^{-\alpha_B T}$ , and  $q_S = qe^{-\alpha_S T}$ . Then  $q_S = q_B\eta^a$ , where  $a = \frac{\alpha_S - \alpha_B}{\lambda}$ . Using  $X$ ,  $p$ ,  $q_B$  and  $\eta$  as decision variables, rewrite problem (A-20) as

$$\max_{X,p,q_B,\eta} \quad r - X - p - q_B - \frac{I[v_B - q_B(1 - \eta)]}{\mu(p + q_B\eta + v_B - r)} \quad (\text{A-21a})$$

$$\text{s.t. } X + p + q_B \eta^a - c_n \geq 0, \quad (\text{I.R. constraint}) \quad (\text{A-21b})$$

$$q_B \eta^a (1 - \eta) < c_n - c_d, \quad (\text{A-21c})$$

$$q_B \eta^a (1 - \eta) + \mu(p + q_B \eta^{a+1}) \geq c_n - c_d, \quad (\text{A-21d})$$

$$X \geq 0, p \geq 0, q_B \geq 0, 0 < \eta \leq 1. \quad (\text{A-21e})$$

The objective function of optimization problem (A-21) is increasing in  $\eta$ . We claim that it is optimal to set  $\eta^* = 1$ . To prove this claim we shall show that if problem (A-21) has a solution, it is feasible to choose  $\eta = 1$ . We shall show this is true by breaking problem (A-21) into two cases.

**Case 1:  $p + q_B > c_n$ .** From Assumption 2,  $c_n \geq \frac{c_n - c_d}{\mu}$ . Therefore, inequality  $p + q_B \geq \frac{c_n - c_d}{\mu}$  (which is the same as condition (A-21d) at  $\eta = 1$ ) is true. Thus, in this case choosing  $\eta = 1$  is feasible and optimal.

**Case 2:  $p + q_B \leq c_n$ .** This implies that for any  $\eta \in (0, 1]$ ,  $p + q_B \eta^a \leq c_n$ . Objective value of problem (A-21) is decreasing in  $X$ . In this case, it is optimal to set  $X^* = c_n - p - q_B \eta^a$ . Problem (A-21) becomes

$$\max_{p, q_B, \eta} r - c_n + q_B(\eta^a - 1) - \frac{I[v_B - q_B(1 - \eta)]}{\mu(p + q_B \eta + v_B - r)} \quad (\text{A-22a})$$

$$\text{s.t. } p + q_B \leq c_n \quad (\text{A-22b})$$

$$q_B \eta^a (1 - \eta) < c_n - c_d, \quad (\text{A-22c})$$

$$q_B \eta^a (1 - \eta) + \mu(p + q_B \eta^{a+1}) \geq c_n - c_d, \quad (\text{A-22d})$$

$$p \geq 0, q_B \geq 0, 0 < \eta \leq 1. \quad (\text{A-22e})$$

Objective value of problem (A-22) is increasing in  $p$ . Therefore, at optimality,  $p = c_n - q_B$ , which we substitute into (A-22) obtaining

$$\max_{q_B, \eta} r - c_n + q_B(\eta^a - 1) - \frac{I[v_B - q_B(1 - \eta)]}{\mu(c_n - q_B + q_B \eta + v_B - r)} \quad (\text{A-23a})$$

$$\text{s.t. } q_B \leq c_n \quad (\text{A-23b})$$

$$q_B \eta^a (1 - \eta) < c_n - c_d, \quad (\text{A-23c})$$

$$q_B \eta^a (1 - \eta) + \mu(c_n - q_B + q_B \eta^{a+1}) \geq c_n - c_d, \quad (\text{A-23d})$$

$$q_B \geq 0, 0 < \eta \leq 1. \quad (\text{A-23e})$$

Objective value of problem (A-23) is increasing in  $\eta$ . At  $\eta = 1$  inequality (A-23d) becomes  $\mu c_n \geq c_n - c_d$ , which is true by Assumption 2. Therefore,  $\eta = 1$  is feasible and optimal.

Thus, we proved that for problem (A-20)  $\eta^* = 1$ . But when  $\eta^* = 1$ , problem (A-20) is equivalent to the inspection problem (10) with  $p + q_S$  replaced by  $p$  and under condition  $p \geq \frac{c_n - c_d}{\mu}$ . This inspection problem solution is given in Proposition 2.  $\blacksquare$

**Proof of Proposition 4:**

For the first part of the proposition, to prove that the buyer prefers the deferred payment mechanism we need to show that  $r - c_n \left(\frac{c_d}{c_n}\right)^{-a} \geq v_B - 2\sqrt{\frac{Iv_B}{\mu}}$ .

From condition  $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ , the buyer's optimal profit with inspection is bounded above by  $v_B - 2\sqrt{\frac{Iv_B}{\mu}} < r - c_n - \sqrt{\frac{Iv_B}{\mu}}$ . Therefore, to prove the dominance of the deferred payment mechanism, it is sufficient to show that  $r - c_n \left(\frac{c_d}{c_n}\right)^{-a} \geq r - c_n - \sqrt{\frac{Iv_B}{\mu}}$ . Equivalently,  $\sqrt{\frac{Iv_B}{\mu}} \geq c_n \left[\left(\frac{c_d}{c_n}\right)^{-a} - 1\right]$ .

From condition  $\frac{I}{\mu} \leq \frac{v_B}{4}$ , it follows that  $v_B \geq 4\frac{I}{\mu}$ , and  $\sqrt{\frac{Iv_B}{\mu}} \geq 2\frac{I}{\mu}$ . Thus, to prove the proposition, it is sufficient to show that  $2\frac{I}{\mu} \geq c_n \left[\left(\frac{c_d}{c_n}\right)^{-a} - 1\right]$ . The last inequality is true by the Proposition hypothesis.

For the second part of the proposition, to prove that the buyer prefers the deferred payment mechanism we need to show that  $r - (c_n - c_d)(1 + a) \left(\frac{a}{1+a}\right)^{-a} \geq v_B - 2\sqrt{\frac{Iv_B}{\mu}}$ .

From condition  $r - c_n > v_B - \sqrt{\frac{Iv_B}{\mu}}$ , the buyer's optimal profit with inspection is bounded above by  $v_B - 2\sqrt{\frac{Iv_B}{\mu}} < r - c_n - \sqrt{\frac{Iv_B}{\mu}}$ . Therefore, to prove the dominance of the deferred payment mechanism, it is sufficient to show that  $r - (c_n - c_d)(1 + a) \left(\frac{a}{1+a}\right)^{-a} \geq r - c_n - \sqrt{\frac{Iv_B}{\mu}}$ . Equivalently,  $\sqrt{\frac{Iv_B}{\mu}} \geq c_n \left[\left(1 - \frac{c_d}{c_n}\right) (1 + a) \left(\frac{a}{1+a}\right)^{-a} - 1\right]$ .

From condition  $\frac{I}{\mu} \leq \frac{v_B}{4}$ , it follows that  $v_B \geq 4\frac{I}{\mu}$ , and  $\sqrt{\frac{Iv_B}{\mu}} \geq 2\frac{I}{\mu}$ . Thus, to prove the proposition, it is sufficient to show that  $2\frac{I}{\mu} \geq c_n \left[\left(1 - \frac{c_d}{c_n}\right) (1 + a) \left(\frac{a}{1+a}\right)^{-a} - 1\right]$ . The last inequality is true by the Proposition hypothesis. ■