Inflation, Money Demand and Portfolio Choice*

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Abstract

We introduce a money demand motive in a life-cycle portfolio choice model and estimate the structural parameters that can generate limited stock market participation and plausible holdings of money, bonds and stocks. The model predicts an increase in bond holdings over the life cycle, and a declining share of money in portfolios as wealth increases. Both predictions are consistent with the data, even though the model overpredicts stock holdings in early life. When mean inflation approaches the Friedman rule, money demand crowds out completely bond demand and the share of financial wealth in stocks decreases.

JEL Classification: E41, G11.

Key Words: Life Cycle Models, Portfolio Choice, Inflation, Money Demand, Stock Market Participation, Uninsurable Labor Income Risk.
1 Introduction

In the recent large literature on portfolio choice\footnote{See Campbell (2006) for a recent excellent survey.} households are assumed to choose between different real assets (typically bonds, stocks and/or housing), ignoring the fact that all transactions in the data are actually done in nominal terms. These models therefore cannot study the effects of inflation on household money demand, portfolio choice and stock market participation decisions. On the other hand, the monetary economics literature starts out with a nominal model and inflation has important real effects and implications for policy. Nevertheless, canonical models in the money demand literature follow the Baumol-Tobin analysis and typically focus on the distinction between money and bonds as a proxy for all other assets in the household portfolio (see, for example, Alvarez and Lippi (2009)).

However, a potentially more important decision (especially for the richest part of the population) involves the asset allocation between transaction-type balances (like money) and stocks (that have a substantially different risk-return trade-off from bonds). Moreover, as documented by Doepke and Schneider (2006), inflation changes the real value of nominal assets causing large redistributitional effects in the macroeconomy. Thus, both the level and volatility of inflation have the potential of affecting household consumption and portfolio choice by changing the real returns of assets.

To build an empirically relevant model of household money demand, we first examine the empirical properties of life-cycle household portfolio choices in three broad asset classes: money (transaction accounts), bonds and stocks. We use both the 2001 and 2007 U.S. Surveys of Consumer Finances (SCF) for this purpose. In the 2001 (2007) SCF 91\% (92\%) of all households had a transactions/liquidity account, 63\% (64\%) had a positive amount of bonds and
52% (50%) participated in the stock market (including participation through retirement plans).

In the SCF data, wealthier households tend to hold a mixture of money, bonds and stocks with money always featuring in the portfolio. Moreover, wealthier households tend to hold less money (around ten to fifteen percent) and more stocks as a percentage of their total financial assets. On the other hand, households that do not participate in the stock market are poorer and tend to hold a very large proportion (more than two thirds throughout their lifecycle) of their financial wealth in money-like, transaction accounts.

To analyze the effect of inflation on the real economy and asset allocation choices we first need to introduce a role for money balances in a model. Introducing money can vary in complexity from the decentralized search Kiyotaki and Wright (1989) setup, to other money demand models, such as cash-in-advance (Lucas and Stokey (1987)), money-in-the-utility function (Sidrauski (1967)) and shopping-time approaches (McCallum and Goodfriend (1987)).

Given that our purpose is to develop a tractable model that can be confronted with the data, we use an approach similar to shopping time models.\textsuperscript{2} Specifically, we assume that money provides liquidity services and therefore a higher amount of money lowers the cost from having to undertake a given transaction for consumption purposes, other things being equal. Everything else we assume is similar to recent life-cycle models that feature intermediate consumption and stochastic uninsurable labor income in the tradition of Deaton (1991) and Carroll (1997)\textsuperscript{3}, and as extended in the life cycle portfolio choice literature by Cocco et al. (2005), for instance. One nice feature of our

\textsuperscript{2}For recent applications of shopping time models, see, for example, Mulligan and Sala-i-Martin (2000).

\textsuperscript{3}Attanasio et al. (1999), Gourinchas and Parker (2002) and Cagetti (2003) extend this tradition and estimate the structural parameters of life cycle models with a single real asset (a riskless bond).
setting is that it nests the life cycle portfolio models where bonds and stocks are real assets and money does not circulate in the economy. We also introduce a fixed cost to participate in the stock market\textsuperscript{4} to generate low stock market participation for one group of households.

Ex ante heterogeneity in risk aversion and the elasticity of intertemporal substitution is used to generate poorer and richer households, as in Gomes and Michaelides (2005). Given the lack of guidance in picking the shopping technology parameters, we estimate these using a method of simulated moments (MSM) estimation technique. We estimate the structural parameters by matching moments from the 2007 Survey of Consumer Finances data and simulated data from the model. Specifically, we match mean financial wealth to labor income over the life cycle for stockholders and non-stockholders, and the portfolio shares across money, bonds and stocks for stockholders and between money and bonds for non-stockholders. We also estimate the one-time fixed cost to generate the 50% participation rate in the 2007 data. The fixed cost is larger than previous estimates (16% of mean labor income) but this is estimated for a relatively high discount factor (0.97)\textsuperscript{5}.

Our model can replicate the fact that the share of wealth in stocks increases as financial wealth rises, which is consistent with the data, while this stylized fact has been a priori inconsistent with recent models of household portfolio choice, as pointed out by Wachter and Yogo (2010). The young (who start out with low financial wealth) have higher liquidity needs and economize on shopping costs by holding liquid balances in the form of money. As financial

\textsuperscript{4}This follows a large recent literature on household portfolio choices to generate limited stock market participation, see for example, Vissing-Jorgensen (2002), Haliassos and Michaelides (2003), Gomes and Michaelides (2005), Alan (2006), Paiella (2007), Attanasio and Paiella (2010), Bonaparte et al. (2012), and Alvarez et al. (forthcoming).

\textsuperscript{5}For a lower discount factor we can estimate a lower cost of participation but we did not want to introduce additional ex ante heterogeneity. For $\beta = 0.95$ we can generate a stock market participation rate of 50% with a fixed cost of 5% of mean annual labor income.
wealth increases over the life cycle, they diversify into stocks and then hold a mixture of all three assets later on in the life cycle.

On the other hand, the model overpredicts (underpredicts) the share of wealth allocated to stocks (bonds) in the early part of the life cycle. The model does relatively well matching quantitatively limited stock market participation, the share of wealth in money, bonds and stocks over all other parts of the lifecycle.

We next use the estimated model to provide answers to interesting counterfactual questions. What are the effects of both the level and volatility of inflation on money demand and asset allocation? Higher mean inflation causes a reallocation away from money into stocks in the model. This effect is stronger for younger agents, who have a stronger demand for stocks in the model and are more likely to increase it when the risk-return trade-off is improved. On the other hand, the portfolio choice of older agents, is relatively insensitive to this change. This arises because older agents have more predictable labor income and therefore their money holdings are stable as a proportion of their financial wealth. Younger households, however, devote a higher share of their financial wealth in liquid balances and therefore are more keen to reallocate out of money in the presence of higher inflation.

Alternatively, introducing deflation to implement the Friedman rule of a zero nominal interest rate, increases dramatically the demand for money. Bonds are completely crowded out from household portfolios. This happens for both stockholders and non-stockholders, and interestingly stock market participation is also substantially reduced under the Friedman rule because money serves both as a medium of exchange as well as a store of value.

What are the hedging demands generated by inflation? We find that hedging demands are small for low to moderate rates (10%) of inflation when in-
flation is perceived to be i.i.d. (as in recent years). We find this surprising but given the level of idiosyncratic uncertainty faced by households, we think it is a reasonable conclusion. Further research can ascertain whether the introduction of a persistent inflation process or money illusion can change this conclusion.

Can inflation have real, aggregate demand effects in the model? Mean inflation affects not only aggregate money demand but also aggregate consumption demand and wealth accumulation. As mentioned above, even though the young substitute money for stocks, the money demand of the old does not change much. The decrease in the real return of money due to higher inflation decreases the overall return of their portfolio and hence decreases their wealth accumulation and consumption.

In terms of the literature, we view the paper as contributing towards understanding money demand and portfolio choice in the presence of nominal assets. Typically, research on money demand focusses on the distinction between money and safe bonds (see, for example, Mulligan and Sala-i-Martin (2000), Alvarez and Lippi (2009)). In our model we make explicit the choice between money (that earns a zero nominal return) and other assets like bonds and stocks that earn the historically observed rates of return. Moreover, we estimate the structural parameters of a life cycle model that can replicate the observed portfolio choices.

The other strand of the literature that the model relates to is the recent life cycle saving and portfolio choice literature (Cocco et al. (2005), Gomes and Michaelides (2005), Polkovnichenko (2007) and Wachter and Yogo (2010) to name some examples). In all these papers, however, the choice is between real assets (real bonds and real stocks) and therefore the effects of inflation on consumption, money demand and portfolio choices cannot be analyzed.
The rest of the paper is organized as follows. Section 2 discusses some stylized facts regarding money holdings over the life cycle. Section 3 presents the model, and Section 4 reports the estimated parameters. Section 5 presents the benchmark numerical results and Section 6 conducts several comparative statics. Section 7 investigates the implications of mean inflation for aggregate money, consumption demand and wealth accumulation. Section 8 concludes.

2 Empirical Evidence on Life Cycle Asset Allocation and Participation in Different Asset Markets

We first need a working definition of money to proceed. We use the same definition that the SCF uses to construct the variable LIQ in the public extract of the data set. Specifically, LIQ is defined as the sum of all checking, saving, money market, deposit and call accounts.

We focus on using a single cross section, the 2007 Survey of Consumer Finances, for establishing the stylized facts about the holdings of money in household portfolios over the life cycle. We have repeated the analysis below for all triennial surveys between 1989 and 2007 and we can report that the results from the 1998, 2001, 2004 and 2007 surveys are very similar along the dimensions we report below. Earlier surveys (the 1989 for example) feature lower stock market participation and higher shares of money in the portfolios. The rise of the equity culture in the 1990s is probably responsible for this change. This points towards having to come up with identifying assumptions to decompose the cross sectional results into age, time and cohort effects in this earlier period, if one is interested in secular changes since 1989. Instead
of following this approach, we compare the results across the 1998 and 2007 surveys and find that our stylized facts are robust both qualitatively and quantitatively across these four surveys. Cohort effects seem to be less important in this period and we therefore interpret the cross-sectional evidence as the set of life cycle facts a good monetary model will need to explain. We therefore leave to future work the time-age-cohort decomposition that could be quite important in understanding the evolution of money demand and stock market participation in the last three decades.

We also restrict our analysis to workers with positive earnings over the year and exclude people with interest in business equity. We focus solely on workers because entrepreneurs have the option of investing in their own business equity (Heaton and Lucas (2000a)) and our labor-income process is based on individual labor income data. In the data, most households have a liquid account to undertake their transactions. In the 2001 (2007) SCF 91% (92%) of all households had a transactions/liquidity account, 63% (64%) had a positive amount of bonds and 52% (50%) participated in the equity market (including participation through retirement plans).

One of the well-known stylized facts in the life cycle portfolio choice literature is that financial wealth is correlated with stock market participation (see Campbell (2006) for a recent survey). We estimate the mean amount of financial wealth for households who hold no stocks but just liquid accounts and bonds (in 2007 terms). The mean (median) amount of financial wealth for this group equals 23,512 (2,020) US$, whereas for the stockholders mean (median) financial wealth equals 290,206 (85,300) US$ illustrating the stark dichotomy between households that hold stocks and households that do not. Table 1 reports the levels of financial assets across the two groups over five broad age categories (four during working life and one during retirement) and
### Table 1: Mean (median) financial wealth for the two main groups (non-stockholders/stockholders) from the 2007 SCF data. The precise definitions for the different variables are in Appendix A.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Median) Non-Stockholders</th>
<th>Mean (Median) Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>6034 (1100)</td>
<td>56966 (20750)</td>
</tr>
<tr>
<td>35-45</td>
<td>10688 (1200)</td>
<td>158989 (62550)</td>
</tr>
<tr>
<td>46-55</td>
<td>21334 (1800)</td>
<td>275634 (106580)</td>
</tr>
<tr>
<td>56-65</td>
<td>31433 (3540)</td>
<td>434306 (155600)</td>
</tr>
<tr>
<td>66+</td>
<td>47852 (6900)</td>
<td>516218 (177200)</td>
</tr>
</tbody>
</table>

Recent work explains this fact using a fixed cost to prevent households from participating in the stock market\(^6\). We also follow the fixed cost approach to generate stock market non-participation in the structural model.

A second well known issue in the literature that comes out from Table 1 is the skewed distribution of financial wealth which affects the choices researchers need to make when bringing models to the data. In general, there are three main mechanisms being used to match the observed wealth distribution: heterogeneous discount rates (Krusell and Smith (1998)), bequests (De Nardi (2004)), and a combination of bequests and entrepreneurship (Castaneda et al. (2003)). These are general equilibrium models with a single asset, whereas we want to eventually solve a model with three different assets and different rates of return. Rather than complicating the model further we abstract from matching the wealth distribution exactly. Instead we focus on matching the evolution of mean financial wealth to labor income over the life cycle. Eventually a general equilibrium model can be calibrated to match these magnitudes.

\(^6\)This follows a large recent literature on household portfolio choices to generate limited stock market participation, see for example, Vissing-Jorgensen (2002), Haliassos and Michaelides (2003), Gomes and Michaelides (2005), Alan (2006), Paiella (2007), Attanasio and Paiella (2010) and Bonaparte et al. (2012).
<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Wealth/Income)</th>
<th>Mean (Wealth/Income)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Stockholders</td>
<td>Stockholders</td>
</tr>
<tr>
<td>20-34</td>
<td>0.31</td>
<td>0.85</td>
</tr>
<tr>
<td>35-45</td>
<td>0.26</td>
<td>1.53</td>
</tr>
<tr>
<td>46-55</td>
<td>0.56</td>
<td>2.65</td>
</tr>
<tr>
<td>56-65</td>
<td>0.77</td>
<td>7.02</td>
</tr>
<tr>
<td>66-75</td>
<td>2.48</td>
<td>15.7</td>
</tr>
</tbody>
</table>

Table 2: Mean financial wealth relative to labor income for the non-stockholders/stockholders in the 2007 SCF data. The definitions for the different variables are in Appendix A.

We leave again the more ambitious task of matching the wealth distribution in the context of this monetary model to future work.

Table 2 reports the moments we will be matching. It contains similar information to that in Table 1 but mean financial wealth is normalized by labor income. There is a jump after retirement as retirement income drops below mean working life labor income.

It should also be noted that we do not include home equity in our measures of household financial wealth: we want to think of the available financial assets and how they are allocated across liquid balances. In the structural model we will use a data-driven housing expenditures life-cycle function to exogenously subtract the expenditures that go in housing from labour income, as is done in Gomes and Michaelides (2005) and Love (2010).

We next go deeper into the role of money in the household portfolio and how money allocations change over the life cycle. We compute the mean asset allo-

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7This is the approach advocated by Heaton and Lucas (2000b). Solving and understanding the intuition behind general equilibrium heterogeneous agent models is difficult. One useful (or intermediate) step involves matching the demand side holding asset returns exogenous and at their observed historical values before a general equilibrium model that matches the data is constructed.
cations across money, bonds and stocks for the households that hold all three assets (stockholders) and also for the households that hold only money and bonds (who we define as non-stockholders). For non-stockholders, the mean share of financial wealth allocated in liquid balances (money) is 66.6% (with a standard deviation equal to 40.8%), and therefore the remainder (33.4%) is allocated in bonds (with a standard deviation equal to 40.8%). For stockholders, we find that the mean share of wealth in stocks is 45.5% (with a standard deviation of 28.6%), the mean share of wealth in money is 17.6% (with a standard deviation of 20.5%) and finally the mean share of wealth in bonds is 36.8% (with a standard deviation of 28.1%).

Table 3 reports the mean portfolio shares for money ($\alpha_m$), bonds ($\alpha_b$) and stocks ($\alpha_s$) for the five age groups and across stockholders and non-stockholders. By definition, non-stockholders hold no equities and we can observe that their portfolios are heavily dominated by the money accounts, with a slightly declining profile over the life-cycle. The life-cycle profiles for stockholders do not show any substantial variations, even though there is a small tendency for the share of wealth in money balances to decrease over the working life cycle and increase after retirement. What is immediately apparent, however, is that money is a key feature of household portfolios, despite the rate of return dominance of other assets, with all age groups devoting a substantial percentage of their financial wealth to money holdings.

3 The Model

The model is a nominal version of life-cycle models that are extensively used in the household portfolio literature. Agents work while they are young, and receive a pension after retirement. They are subject to uninsurable labor
Life Cycle Portfolio Choice

<table>
<thead>
<tr>
<th>Age Group</th>
<th>( \alpha_m )</th>
<th>( \alpha_b )</th>
<th>( \alpha_s )</th>
<th>( \alpha_m )</th>
<th>( \alpha_b )</th>
<th>( \alpha_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>75.8</td>
<td>24.2</td>
<td>0.0</td>
<td>28.6</td>
<td>29.8</td>
<td>41.5</td>
</tr>
<tr>
<td>35-45</td>
<td>67.9</td>
<td>32.1</td>
<td>0.0</td>
<td>16.7</td>
<td>35.3</td>
<td>48.0</td>
</tr>
<tr>
<td>46-55</td>
<td>62.5</td>
<td>37.5</td>
<td>0.0</td>
<td>14.4</td>
<td>37.6</td>
<td>48.0</td>
</tr>
<tr>
<td>56-65</td>
<td>59.2</td>
<td>40.8</td>
<td>0.0</td>
<td>13.5</td>
<td>38.7</td>
<td>47.8</td>
</tr>
<tr>
<td>66-75</td>
<td>63.3</td>
<td>36.7</td>
<td>0.0</td>
<td>16.2</td>
<td>42.0</td>
<td>41.8</td>
</tr>
</tbody>
</table>

Table 3: Mean financial portfolios for the non-stockholders/stockholders in the 2007 SCF data. The definitions for the different variables are in Appendix A.

income risk and borrowing constraints. There are three assets in the economy, money, bonds and stocks, and they are traded in nominal terms. In order to introduce money, we extend the model by introducing nominal assets and transaction frictions.

### 3.1 Preferences

Time is discrete and \( t \) denotes adult age which, following the typical convention in the literature, corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 (\( T \)) periods (age 100). The probability that a consumer/investor is alive at time \( (t + 1) \) conditional on being alive at time \( t \) is denoted by \( \xi_t \) (\( \xi_0 = 1 \)). Finally, the consumer/investor has a bequest motive.

Households have Epstein-Zin-Weil utility functions (Epstein and Zin (1989), Weil (1990)) defined over one single non-durable consumption good. Let \( C_{i,t} \) and \( X_{i,t} \) denote respectively real consumption and nominal wealth (cash on hand) of agent \( i \) at time \( t \). Then the real cash on hand is defined as \( X_{i,t}/P_t \) where \( P_t \) denotes the price level at time \( t \). The preferences of household \( i \) are
defined over real consumption by

\[
V_{it}^j \left( \frac{X_{it}}{P_t} \right) = \left\{ (1 - \beta)C_{it}^{1-1/\psi_j} + \beta \left[ E_t \left[ \xi_j V_{it+1} \left( \frac{X_{it+1}}{P_{t+1}} \right)^{1-\rho_j} \right] + (1 - \xi_j)\varphi \left( \frac{X_{it+1}}{P_{t+1}} \right)^{1-\rho_j} \right] \right\}^{\frac{1}{1-1/\psi_j}}
\]

where \( \rho \) is the coefficient of relative risk aversion, \( \beta \) is the discount factor, and \( \varphi \) determines the strength of the bequest motive. Following Vissing-Jorgensen (2002) we assume that different households are heterogeneous in their inter-temporal elasticity of substitution, \( \psi \). Our economy is populated with two equally-sized groups \( J \), respectively, with high (\( \psi_H \)) and low (\( \psi_L \)) intertemporal elasticity of substitution. We also utilize ex ante heterogeneity in risk aversion (high (\( \rho_H \)) and low (\( \rho_L \))) following Gomes and Michaelides (2005) to generate a stronger variation in precautionary saving motives over the life cycle (and therefore more variation in wealth accumulation profiles).

### 3.2 Labor Income Process

Following the standard specification in the literature, the labor income process before retirement is given by

\[
Y_{i,t} = Y_{i,t}^p U_{i,t}
\]

\[
Y_{i,t}^p = \exp(f(t, Z_{i,t}))Y_{i,t-1}^p N_{i,t}
\]

where \( f(t, Z_{i,t}) \) is a deterministic function of age and household characteristics \( Z_{i,t} \), \( Y_{i,t}^p \) is a permanent component with innovation \( N_{i,t} \), and \( U_{i,t} \) a transitory component. We assume that \( \ln U_{i,t} \) and \( \ln N_{i,t} \) are independent and identically distributed with mean \( \{-0.5 * \sigma^2_u, -0.5 * \sigma^2_n\} \), and variances \( \sigma^2_u \) and \( \sigma^2_n \), respec-
tively. The log of $Y_{i,t}$ evolves as a random walk with a deterministic drift, $f(t, Z_{i,t})$. For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period $K$, corresponding to age 65 ($K = 46$). Earnings in retirement ($t > K$) are given by $Y_{i,t} = \lambda Y_{i,K}^{p}$, where $\lambda$ is the replacement ratio ($\lambda = 0.68$).

Durable goods, and in particular housing, can provide an incentive for higher spending early in life. We exogenously subtract a fraction of labor income every year allocated to durables (housing). This empirical process is taken from Gomes and Michaelides (2005) and is based on PSID data.

3.3 Specification of shopping cost technology

In order to motivate money holdings, we assume transaction frictions. Our approach is related to shopping time models, first proposed by McCallum and Goodfriend (1987). We modify the model in order to incorporate it more easily in the portfolio choice literature. In shopping time models, transaction costs are modeled in terms of foregone time: money can help reduce transaction time. As is shown in Lucas (2000), there is a connection between the shopping time models and the inventory-theoretic studies of money (Baumol (1952), Tobin (1956)). More broadly speaking, the transaction cost can include not only a shopping cost but also a cost of selling illiquid assets to finance consumption. Different versions assume different trade-offs in the presence of transactions frictions. For example, Lucas (2000) assumes that agents face a trade-off between hours spent on production and transactions. Ljungqvist and Sargent (2004) (Ch. 24) assume a trade-off between transaction time and leisure.

To generate money holdings, we assume a shopping cost transaction friction

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8See Alvarez and Lippi (2009) for recent developments.
(proportional to $Y_{it}^p$), a direct physical cost in consumption goods:

$$
\Omega_{it} Y_{it}^p = \Omega(C_{it}, M_{it}/P_t; \varepsilon, \kappa) Y_{it}^p, \quad \Omega_C > 0, \quad \Omega_M < 0
$$

The cost is increasing in real consumption and decreasing in real money balances. In the benchmark case we assume

$$
\Omega_{it} Y_{it}^p = \varepsilon \left( \frac{C_{it}}{M_{it}/P_t} \right)^\kappa Y_{it}^p, \quad \varepsilon > 0, \kappa > 0
$$

(4)

It will be convenient later on to divide variables by the permanent component of labor income. In this case, the shopping cost per unit of $Y_{it}^p$ is given by

$$
\Omega_{it} = \varepsilon \left( \frac{c_{it}}{m_{it}} \right)^\kappa, \quad \varepsilon > 0, \kappa > 0
$$

(5)

where $c_{it} = \frac{C_{it}}{Y_{it}^p}$ and $m_{it} = \frac{M_{it}}{Y_{it}^p P_t}$. Our preferred interpretation is that the transaction cost represents an opportunity cost of time which is why we assume that it is proportional to the permanent component of labor income. The functional form (4) is consistent with Lucas (2000) who shows that the implied money demand function is consistent with the demand function of Baumol (1952) and Tobin (1956). The parameter $\varepsilon$ measures the severity of transaction frictions. A large $\varepsilon$ means it takes more resources to do transactions and it can be different over the life cycle or across agents.

We model transaction costs as a direct physical cost in terms of consumption goods. An advantage of our approach is that we can treat money by exactly the same way as we treat bonds and stocks because there is no additional margin between money holding decisions and leisure (or labor supply) decisions. Therefore our model maintains the basic structure of the models used in the portfolio choice literature, making the model computationally tractable.
and making the results easily comparable to those obtained in the literature. The presence of the permanent component of income in our shopping cost formulation (4) is consistent with the spirit of the shopping time technology specification which relates the cost of illiquidity to the wage rate faced by the household. Also, our modeling approach maintains the basic properties of the shopping time models — money demand will be increasing in consumption and decreasing in nominal interest rates.

3.4 Financial Assets and Constraints

The agent has options to hold three kinds of assets: fiat money \( M_{i,t} \), nominal bonds \( B_{i,t} \) and nominal stocks \( S_{i,t} \). We let \( X_{i,t} \) be nominal “cash on hand” that the agent can use for consumption and portfolio decisions. The budget constraint is given by

\[
X_{i,t} = P_tC_{i,t} + S_{i,t} + B_{i,t} + M_{i,t}. \tag{6}
\]

We assume that the shopping cost is deducted at the beginning of the next period. This timing assumption ensures that \( X_{i,t} \) is a state variable, as in the portfolio choice literature. Then, the evolution of \( X_{i,t} \) is given by

\[
X_{it+1} = R^a_{t+1}S_{it} + R^b_{t+1}B_{it} + M_{it} + P_{t+1}Y_{it+1} - P_{t+1}\Omega_{it}Y^p_{it} - 1_t(.)P_{t+1}FY^p_{it} \tag{7}
\]

where \( R^a_{t+1} \) and \( R^b_{t+1} \) respectively denote the nominal returns of stocks and bonds. Note that the nominal return of fiat money is unity. Finally, \( Y_{i,t+1} \) is real income at time \( t + 1 \). The indicator function \( 1_t(.) \) becomes one when the fixed cost to participate in the stock market is incurred.\(^9\)

\(^9\)We deviate slightly from the rest of the literature by assuming that the cost is paid next period. The results are robust to assuming that the cost is incurred in the current period.
Finally, as in the portfolio choice literature, we prevent households from borrowing against their future labor income. More specifically we impose the following restrictions:

$$B_{i,t} \geq 0$$
$$S_{i,t} \geq 0$$
$$M_{i,t} \geq 0$$

We have one continuous state variable: $X_{i,t}$ and the control variables are $C_{i,t}$, $M_{i,t}$, $S_{i,t}$ and $B_{i,t}$.

### 3.5 Normalizing by Prices and Growth

Let lower case letters denote real variables normalized by the permanent component of labor income ($Y_{i,t}^p$). For example the normalized real cash on hand is defined as $x_{i,t} = X_{i,t}/(Y_{i,t}^pP_t)^{10}$. The evolution of the state variable is then given by

$$x_{i,t+1} = \frac{r_s^{i+1}}{g_{i,t+1}} s_{i,t} + \frac{r_b^{i+1}}{g_{i,t+1}} b_{i,t} + \frac{r_m^{i+1}}{g_{i,t+1}} m_{i,t} + U_{i,t+1} - \frac{\Omega_{i,t}}{g_{i,t+1}} - \frac{1}{g_{i,t+1}} f_{i+1}$$

where

$$r_s^{i+1} \equiv R_{t+1}^s \pi_{t+1}^{-1}, \quad r_b^{i+1} \equiv R_{t+1}^b \pi_{t+1}^{-1}, \quad r_m^{i+1} \equiv \pi_{t+1}^{-1}$$

are respectively the real returns of stocks, bonds and money, where $\pi_{t+1} \equiv P_{t+1}/P_t$ denotes gross inflation, and $g_{i,t+1} \equiv Y_{i,t+1}^p/Y_{i,t}^p$ is the gross growth rate of the permanent component of labor income.

The representation of consumer preferences in terms of stationary (normal-

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10 The only lower-case variable not normalized by $P_t$ is consumption ($C_{i,t}$) which was in real terms from the beginning.
ized) units is given in Appendix B.

4 Parameter Estimation

We estimate the structural model using the method of simulated moments (MSM). Given the large number of parameters in the model we will calibrate certain parameters and then estimate the fixed cost, shopping cost and bequest preference parameters, since these are the parameters for which there is no clear guidance in the literature on how to pick them. The calibration for labor income uses the estimates in Cocco et al. (2005) so that $\sigma_u = 0.1$, $\sigma_n = 0.08$, and $\lambda = 0.68$. Given the positive correlation between education, financial wealth and the probability to participate in the stock market, we use the hump shape process for households with a college degree from Cocco et al. (2005).

We will use exogenous processes for stock and bond returns, inflation and the aggregate component of labor income. Given that we calibrate the cross sectional model to decisions taken in 2007, we use the period 1995 to 2008 to compute descriptive statistics and correlations between these variables. In section 6.4 we provide comparative statics experiments later on based on historical experience. The table below reports the descriptive statistics for the variables of interest.

We also assume an i.i.d process for stock returns with a mean real return equal to six percent and a standard deviation equal to 22%. The bond return process is similarly calibrated with a mean return equal to two percent and a standard deviation equal to three percent.

We also need to take a stance on the correlations across these variables. These are shown in Table 5. Based on our data sample, we set the correlations
### Means and Standard Deviations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>2.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Bond Returns</td>
<td>2.4</td>
<td>2.6</td>
</tr>
<tr>
<td>Stock Returns</td>
<td>6.8</td>
<td>22.0</td>
</tr>
<tr>
<td>Wage growth</td>
<td>2.7</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 4: We report the means and standard deviations (S.D.) of key inputs in the decision model. All variables are real, and the bond return is the return on the one-year bond. Details about the data can be found in Appendix A.

### Correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inflation</th>
<th>Bond Returns</th>
<th>Stock Returns</th>
<th>Wage growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond Returns</td>
<td>-0.5</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock Returns</td>
<td>0.25</td>
<td>0.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Wage growth</td>
<td>0.0</td>
<td>0.4</td>
<td>0.4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 5: We report the correlation matrix of key inputs in the decision model. All variables are real, and the bond return is the return on the one-year bond. This is for the period between 1995 and 2008. Details about the data can be found in Appendix A.

between bond and stock returns, as well as the correlation between inflation and the real wage growth, to zero.

To use the method of simulated moments we need to decide which moments to match. The key variables of interest for our purposes are the mean holdings of financial wealth over the life cycle for stock holders and non-stockholders and the mean participation rate. Conditional on the participation status we then have asset allocations between money, bonds and stocks sorted by age, and for non-stockholders the allocations between bonds and money. This gives a total of twenty six moment conditions.

Given the large number of preference parameters we follow the empirical evidence in Vissing-Jorgensen (2002), the calibration in Gomes and Michaelides (2005) and the estimation in Gomes et al. (2009) and use two different values for the elasticity of intertemporal substitution, higher for the households more
Calibrated Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\psi_H$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\psi_L$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho_H$</td>
<td>5.0</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>1.2</td>
</tr>
<tr>
<td>$\sigma_U$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6: Calibrated structural parameters.

likely to participate in the stockmarket and lower for the rest. We also utilize heterogeneity in risk aversion rather than discount rates to generate low propensities to save, thereby generating poorer households who are therefore less likely to incur the fixed stock market participation cost. In our baseline estimation we choose $\psi_H = 0.6$, $\psi_L = 0.1$, $\rho_H = 5.0$ and $\rho_L = 1.2$.

Our preference parameters for non-stockholders might appear strange given that they have lower risk aversion (on average) than stockholders. The incentive to save, however, is what matters in incurring the fixed cost to participate in the stock market and this is monotonically related to the precautionary savings motive (Haliassos and Michaelides (2003)). Our calibration is consistent with the low risk aversion parameters for non-participants found in Alan (2006), Paiella (2007) and Attanasio and Paiella (2010).

For compactness purposes the next table lists all calibrated parameters, which include an annual discount factor at 0.97, and the standard deviations of the permanent and transitory labor income shocks as used by Carroll (1997) and Cocco et al. (2005).

We then estimate the remaining structural parameters for which there is much less guidance from the empirical literature. These are the two shopping cost parameters $\{\varepsilon, \kappa\}$, the bequest parameter $\varphi$ and the fixed cost parameter.
There is some empirical evidence on the size of the bequest parameter (De Nardi (2004)) but given our relatively simple retirement specification, it is not clear our results should be compared with those and that is also a reason for choosing to estimate this preference parameter as well. On the size of the fixed cost in the context of this model, the closest paper is Alan (2006) and our non-stockholders would be similar to the non-participants modelled through a fixed cost in that paper.

5 Results

The estimated parameters for our model are given in table 7. The results are consistent with previous estimates in the literature. There is some evidence for a bequest motive needed because financial wealth is not fully decumulated during retirement, this is consistent with, among others, De Nardi (2004). There are no micro estimates for the equivalents of the shopping cost parameters against which we can compare our results (this was also one of the reasons for performing structural estimation). The implied shopping cost varies between 0.5 and 2.0 percent of mean annual labor income that we view as a reasonable

\[ D = \left( \frac{1}{T} \sum_{t=1}^{T} \text{moments}(Y_t) - \frac{1}{TH} \sum_{t=1}^{TH} \text{moments}(\tilde{Y}_t) \right). \]

where moments() denotes a particular moment. The asymptotically efficient optimal weighting matrix $S^{-1}$ equals the inverse of the variance-covariance matrix of the data. Following Appendix B in De Nardi et al. (2010), we use a diagonal weighting matrix for $S^{-1}$ with the elements along the diagonals being the variance of each moment from the data.
### Estimated Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>0.72</td>
<td>0.02</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.018</td>
<td>0.003</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.54</td>
<td>0.01</td>
</tr>
<tr>
<td>$F$</td>
<td>0.16</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 7: Estimated structural parameters. Standard errors are computed through numerical derivatives.

transaction cost and is consistent with Lucas (2000).

The fixed cost to generate non-participation deserve some discussion. The closest paper to our setup is Gomes and Michaelides (2005) who calibrate a smaller cost to generate non-participation. We can replicate their results if we reduce the mean real return on the bond from 2.4% to 2.0% and decrease the discount factor to 0.95 (which is what they use). We opted to limit ex ante heterogeneity and therefore have a higher fixed cost. Our view is that the higher cost should be interpreted as a short cut for anything ranging from inertia, behavioral biases, low trust in the stock market (Guiso et al. (2008)), observation and transaction costs stemming from rational inattention (Alvarez et al. (forthcoming)), or repeated costs from having a stock trading account (small annual trading costs can add up over a few years as in Bonaparte et al. (2012)).

What are the policy functions and life cycle profiles implied by these parameter estimates? Figure 1 shows the portfolio choice policy functions of the low risk aversion- low EIS group against the continuous state variable (cash on hand, $x$). We show the policy functions for the young (age 25), middle-aged (age 55) and retirees (age 85). The solid lines show the portfolio choices of households that have already paid the fixed cost to participate in the stock market. The dashed lines show the choices of non-participants. The vertical axis plots portfolio shares as a percentage of financial wealth invested in each
asset (between zero and one due to the no borrowing/no short sale constraints). The three age groups mainly hold money when their cash on hand is small. Especially, the young and the middle-aged agents invest almost their entire assets in money when they are poor. This is consistent with the data that shows that the young and poor agents tend to hold more money. In line with the portfolio literature, the bond share is increasing in cash on hand while the stock share is decreasing in cash on hand.

The dashed lines indicate the policy functions for the non-participants. These households need to accumulate assets in money and bonds before incurring the fixed participation cost to invest in the stock market. They therefore accumulate money first (to minimize the shopping cost), and then bonds and then if they accumulate a sufficient amount of wealth to justify incurring the fixed participation cost, they do so. At that point they reduce their bond holdings and invest in the stock market.

A similar picture arises from Figure 2 that plots the portfolio policy functions for the higher risk aversion - higher EIS group. The first difference from the previous figure is that the participation takes place faster even though these households are more risk averse. Because of a stronger precautionary saving motive they accumulate more saving and have a stronger incentive to participate in the stock market. The second difference is that the share of wealth in stocks is more quickly declining as a function of cash on hand again because of the higher risk aversion.

Figure 3 conditions on participation status (some of the low risk aversion households will have had lucky labor income draws and participate in the stockmarket) and shows the simulated paths of consumption, financial wealth and income over the life cycle. We simulate the model economy with 10,000 individuals in each age cohort starting with the initial financial wealth taken
from the 2007 SCF and take the mean of each variable. The non-stockholders
do not accumulate much financial wealth, and their consumption tracks la-
bor income over the life-cycle. The stockholders accumulate a higher amount
of financial wealth and therefore their consumption is decoupled from labor
income.

Finally, Figure 4 shows simulated portfolio choices over the life cycle. The
young non-stockholders hold mainly money, and the money share decreases as
the agents become older, reaching a minimum at retirement. At that point
money holdings increase again as wealth is decumulated during retirement. To
minimize on shopping costs during retirement, money gets accumulated in the
portfolio and bonds are crowded out.

Stock market participants are richer households who invest heavily in stocks
from the beginning of working life. Nevertheless, contrary to traditional port-
folio choice models they do not invest everything in stocks because a certain
minimum of money needs to be held to minimize shopping costs. It should be
noted that the share of wealth allocated to stocks increases over certain age
ranges as households get richer. This is a prediction that the real model typi-
cally cannot generate unless extended in other directions (Wachter and Yogo
(2010), for example).

Figure 5 plots the profile of the stock market participation rate. This is
increasing over the lifecycle, consistent with the wealth accumulation motive.
The average stock participation rate implied by the model is 53.2% versus
50.3% in the 2007 SCF data.

How do other predicted moments compare with the actual ones? We first
go through the mean wealth to mean labor income ratios which are given
in Table 8. We observe that the model predicts some underaccumulation in
wealth levels relative to the financial wealth present in the data. Overall, given
Mean Financial Wealth/Income: Data vs Model

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Non-Stockholders</th>
<th>Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>20-34</td>
<td>0.31</td>
<td>0.16</td>
</tr>
<tr>
<td>35-45</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>46-55</td>
<td>0.56</td>
<td>0.54</td>
</tr>
<tr>
<td>56-65</td>
<td>0.77</td>
<td>0.76</td>
</tr>
<tr>
<td>66-75</td>
<td>2.48</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Table 8: Actual versus predicted moments for mean financial wealth relative to mean labor income for the nonstockholders/stockholders. The model is compared to the 2007 SCF data. The definitions for the different variables are in Appendix A

Life Cycle Portfolio Choice by Age

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Non-Stockholders</th>
<th>Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>20-34</td>
<td>75.8</td>
<td>98.3</td>
</tr>
<tr>
<td>35-44</td>
<td>67.9</td>
<td>97.2</td>
</tr>
<tr>
<td>45-54</td>
<td>62.5</td>
<td>68.0</td>
</tr>
<tr>
<td>55-64</td>
<td>59.1</td>
<td>53.3</td>
</tr>
<tr>
<td>65+</td>
<td>63.3</td>
<td>70.2</td>
</tr>
</tbody>
</table>

Table 9: Actual versus predicted moments for mean financial portfolios for the nonstockholders. The model is compared to the 2007 SCF data. The definitions for the different variables are in Appendix A

the cross-sectional uncertainty in the data, we view these profiles as reasonable.

We next present the moments for the portfolio shares. We start with the profiles for non-stockholders, given in Table 9. The model predicts a higher share of wealth in money in the early parts of the life cycle (ages 20-44) relative to the data, but the implications are consistent with the data from that point onwards. In particular, the share of wealth in bonds is rising over the life cycle in both data and model and the quantitative magnitudes are similar from around age 45 onwards.

Next we move on to the comparison between data and model for stockholders. Again, the main failure of the model is that it cannot match bond-holdings
<table>
<thead>
<tr>
<th>Age Group</th>
<th>Data $\alpha_m$</th>
<th>Model $\alpha_m$</th>
<th>Data $\alpha_b$</th>
<th>Model $\alpha_b$</th>
<th>Data $\alpha_s$</th>
<th>Model $\alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>28.6</td>
<td>35.7</td>
<td>29.9</td>
<td>0.0</td>
<td>41.5</td>
<td>64.3</td>
</tr>
<tr>
<td>35-44</td>
<td>16.7</td>
<td>17.3</td>
<td>35.3</td>
<td>3.4</td>
<td>48.0</td>
<td>79.3</td>
</tr>
<tr>
<td>45-54</td>
<td>14.4</td>
<td>11.9</td>
<td>37.6</td>
<td>32.7</td>
<td>48.0</td>
<td>55.3</td>
</tr>
<tr>
<td>55-64</td>
<td>13.5</td>
<td>12.2</td>
<td>38.7</td>
<td>38.4</td>
<td>47.8</td>
<td>49.4</td>
</tr>
<tr>
<td>65+</td>
<td>16.2</td>
<td>19.2</td>
<td>42.0</td>
<td>39.5</td>
<td>41.8</td>
<td>41.3</td>
</tr>
</tbody>
</table>

Table 10: Actual versus predicted moments for mean financial portfolios for the stockholders. The model is compared to the 2007 SCF data. The definitions for the different variables are in Appendix A.

for the first two age groups. Equivalently, as in the real model without time-varying investment opportunities, the model overpredicts the share of wealth invested in the stock market. Specifically, the share of wealth in stocks is 64.3% versus 41.5% in the data for the 20-34 age group, and for the next age group (35-44) this share rises to 79.3% in the model versus 48.0% in the data. Note, however, that the share of wealth in stocks can be rising according to the model because of the decline in the share of money as financial wealth is accumulated. This is an implication that cannot be generated in the special case of this model when money does not circulate in the economy.

The results illustrate the strong demand for stocks early in life as labor income is mostly seen like a riskless asset. Nevertheless, money is held in the portfolio for transaction purposes, especially for the non-participants in the stock market. Moreover, the share of wealth in money does drop over the life cycle for both groups, both in the model and in the data. Note that the presence of money does change the composition of the portfolio relative to other models in the portfolio choice literature that lump money and bonds in the same category. Specifically, in these models the standard prediction is that stockholders should allocate all financial wealth in stockholding while in this
setup the very young (ages 25-44) allocate between 64 to 79 percent of their financial wealth in stocks. The allocation to bonds is still underpredicted relative to the data but we view the model as getting one step closer to matching observed behavior in the data.

6 Comparative Statics

6.1 No money: No Shopping and No Participation Cost

To understand the predictions of the model better, we next perform a series of comparative statics. The first model we can compare our results to is the standard portfolio choice model with no stock market participation costs and where money does not circulate. In our model this specification is nested by setting the shopping technology parameter \( (\varepsilon) \) equal to zero and the fixed cost of participation \( (F) \) also equal to zero. Figure 6 compares this special case (dotted line) with our baseline estimated model. This is an interesting case because money does not circulate in this economy and the model becomes identical with the recent models on household portfolio choice like Cocco et al. (2005).

We can see that the models that treat money and bonds as perfect substitutes generate a large demand for stocks early in life because future labor income is treated like a bond: all saving is done through the stock market. On the other hand, the shopping technology model generates a demand for money that produces an upward sloping share of wealth in stocks over some of the early years of the lifecycle and also reduces substantially the demand for stocks early in life. The demand for bonds is almost unaffected.

The model without money clearly predicts that the share of wealth in stocks decreases as financial wealth rises, a counterfactual prediction. Re-
cently Wachter and Yogo (2010) argue that non-separabilities in the utility function across different goods can generate the upward sloping shape for the share of wealth in stocks as financial wealth increases. Our model provides an alternative explanation that relies on the determinants of money demand and treating transaction accounts and bond investments as assets with different risk/return characteristics.

6.2 Effects of mean inflation

Figure 7 shows the results when mean inflation is substantially increased (we set annual inflation equal to ten percent). We present results separately for stockholders and non-stockholders. A high mean inflation decreases the mean rate of return of holding money, and as a result, households reduce money holdings, which is in line with the money demand literature. Interestingly, Figure 7 shows that the portfolio choice of young agents is particularly affected by inflation, but older agents do not change their portfolio choice significantly. This arises because older agents have more predictable streams of income and therefore their money holdings are stable as a proportion of their financial wealth. Younger households, however, devote a higher share of their financial wealth in liquid balances and therefore are more keen to reallocate out of money in the presence of higher inflation. Another interesting implication of the model is that younger households hedge inflation by reallocating their money holdings towards stocks rather than bonds, in contrast to the money demand literature that focuses on the choice between money and bonds.\footnote{This is also related with the so-called Tobin effect that argues that inflation has potential of inducing capital accumulation because agents substitute money for stocks. It would be interesting to analyze this prediction more fully in a general equilibrium setting.}

Figure 8 illustrates how the behavior of non-stockholders is affected by higher inflation. Interestingly, there is a substantially stronger effect in the
re-allocation from money into bonds for this group than for the stockholders (as a proportion of total financial wealth). Moreover, this tends to happen later in life when this group has had some time to accumulate some wealth (albeit small) for retirement.

Figure 9 shows that higher inflation increases slightly stock market participation as this takes place near retirement when these households have accumulated some savings and they then see the need to hedge higher mean inflation by investing in the stock market.

6.3 The Friedman Rule

We next examine the implications of the Friedman rule on saving and portfolio choices. The Friedman rule is defined as a zero mean nominal interest rate, which implies a mean inflation of -2.4% given our choice of the mean real return on bonds of 2.4%. We again show first the results for stockholders. In this case, the mean returns on bonds and money become almost identical. As is shown in Figure 10, the demand for money significantly increases, in line with the literature. What is striking is the fact that bonds are perfectly substituted out by money. Given the large increase in the share of wealth allocated to money generated by the deflationary expectations, there is a substantial decrease in the demand for stocks by almost all cohorts.

Figure 11 shows how non-stockholders change their behavior in a deflationary environment. Consistent with the behavior of stockholders, there is a large increase in money demand. The increase is sufficiently large to crowd out completely the demand for bonds throughout the lifecycle. Figure 12 illustrates that there is a substantial decrease in the stock market participation rate during the deflationary period because money now provides a better rate of return to its holders.
6.4 Hedging Demands

We next examine how various volatilities and correlations of shocks affect portfolio choice over life cycle. When mean inflation is low at 2.5% as in the benchmark case, we find that increasing inflation volatility has little effects on portfolio choice. More specifically, increasing the standard deviation of inflation by 5 percentage points, even with mean inflation at 2.5% does not have significant effects on household behavior.

We also examine the effects of changing the correlation of inflation with bonds and stock returns through two experiments. First we set the correlation of inflation with stock returns equal to zero, keeping the other correlations the same as in the benchmark case. In the second, we set the correlation of inflation with bond returns equal to zero. Both experiments show that those correlations have negligible effects. Overall, we found that when inflation is low at 2.5%, hedging demands due to inflation volatility are very small. One hypothesis might be that hedging demands due to volatile inflation may be important only when inflation is high. Data show that inflation volatility is positively correlated with the mean inflation rate. For this purpose, we also consider the economy with a mean inflation rate at 10%, and examine the effects of higher inflation volatility in this regime. Comparing the cases in which the standard deviation of inflation is 1% and 5% respectively barely affects portfolio choice. In the interest of space we do not report those figures.

What is the intuition behind these results? The intuition is that most risk households face is idiosyncratic and not aggregate. The volatilities of aggregate risks (wages and inflation) are an order of magnitude lower than idiosyncratic labor income risk. Even very high correlations between aggregate variables do not translate to individual hedging demands exactly because they have a relatively small effect on the idiosyncratic risks households face.
This points towards building models with stronger persistence in the aggregate variables and/or models with long-run co-integrating relationships between aggregate variables (like, for instance, Benzoni et al. (2007)). Such long run co-integrating relationships (for instance between the aggregate component of labor income and dividends) can increase the risk of stock market investments and further reduce the demand for stocks early in life. At the same time they might generate stronger hedging demands. We leave this interesting and important extension for future research.

7 Implications for Aggregate Money Demand, Consumption and Wealth

What is the partial equilibrium relationship between money demand and the nominal interest rate on bonds? Figure 13 shows aggregate money demand, consumption and wealth accumulation against different values of the mean nominal interest rate.\(^\text{13}\) Consistent with the literature (see ?), aggregate money demand is decreasing in the nominal interest rate, and increases rapidly as the nominal rate approaches zero. Note that as the nominal interest rate changes in the figure, we do not keep aggregate consumption constant. Therefore, the change in money demand comes both from the increase in the nominal interest rate and from endogenous changes in consumption.

Aggregate consumption also decreases as the nominal rate increases. Our model implies that aggregate consumption demand decreases by some 6\% as the nominal rate increases from zero to 12.4\%. This is due to the decrease in aggregate wealth, which is also shown in the figure. Aggregate wealth

\(^{13}\)We calibrate the mean net real return on bonds as 2.4\%, so the mean net nominal interest rate shown in the Y-axis of Figure 13 is the mean net inflation plus 0.024.
accumulation decreases by the same magnitude as consumption. The decrease in wealth is much larger than the increase in the shopping cost. Even when the nominal interest rate is 12.4%, the flow shopping cost per permanent income is on average 1.78%. In Lucas (2000), the shopping cost per income when the nominal interest rate is 12.4% can be computed as 1.76%, which is very similar to ours.

Unlike the standard shopping time models, the labor income process is exogenous in this framework. Therefore, the decrease in wealth is mainly due to the effects of inflation on agents’ consumption-saving and portfolio decisions. As mentioned in Section 6, the young substitute money for stocks when inflation rises. Therefore the wealth of the young does not necessarily decrease when inflation rises. However, the portfolio choice of the old is relatively insensitive to inflation unless inflation is close to the Friedman rule. Then the decrease in real return of money due to high inflation decreases the overall return of their portfolio and hence decreases their wealth accumulation and consumption.

8 Conclusion

We estimate the preference parameters of a life cycle money demand and portfolio choice model. The predictions of the model are consistent with the data on limited stock market participation, money demand and wealth accumulation over the lifecycle but the model overpredicts allocations to the stock market in the early period of life (and therefore underpredicts bond holdings until age 44). We use the model to analyze how inflation or deflation affects money demand, asset allocation and stock market participation over the lifecycle.
The hedging demands generated by the model are small: future work can add time-varying investment opportunities to investigate how results might change. Future work can also extend the analysis in a general equilibrium setting to address policy questions such as the effects of changes in the relative supply of money and bonds through open market operations by the central bank. Relatedly, a general equilibrium model will be more useful in computing the welfare cost of inflation across different households, and in the aggregate.

Appendix A  The Data

A.1 Survey of Consumer Finances

We use repeated cross sections from the U.S. Survey of Consumer Finances to establish certain robust facts with regards to household choices across liquid accounts (money), bonds and stocks. Total financial assets are broken up into the three broad categories the model has implications for: liquid resources (LIQ), stock (EQUITY) and nonequity (BOND) investments. In the 2001 public extract of the SCF data set, LIQ is defined as the sum of all checking, saving, money market deposit and call accounts. We follow the same convention and LIQ becomes our measure of money when confronting the model implications to the data. EQUITY is defined in the same extract as all financial assets invested in stocks and this comprises the following categories:

1) directly held stock

2) stock mutual funds (the full value is assigned if the fund is described as a stock mutual fund, and half the value for combination mutual funds)

3) IRAs/Keoghs invested in stock (full value if mostly invested in stock, half value if split between stocks/bonds or stocks/money market, one third value if split between stocks/bonds/money market),

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4) other managed assets with equity interest (annuities, trusts, MIAs) (where again the full value is used if mostly invested in stock, half value if split between stocks/MFs & bonds/CDs, or "mixed/diversified," and one third value if "other")

5) thrift-type retirement accounts invested in stock (full value if mostly invested in stock and half value if split between stocks and interest earning assets) and

6) savings accounts classified as 529 or other accounts that may be invested in stocks. We classify the remaining financial assets as BOND and interpret them as capturing the bond investments in the model (both government and corporate bonds are lumped together in this category).

A.2 Aggregate Data

We use the CRSP data base to download annual US inflation, bond and stock returns from 1925 to 2008. We use the 1995-2008 averages because we focus on matching the data from the 2007 SCF survey but report results for other samples as well. For the aggregate component of labor income we use the NIPA wages and salary disbursement series and we deflate using the inflation rate from CRSP.

Appendix B The Normalized Value Function

Let \( v_{i,t}^J = V_{i,t}/Y_{i,t}^p \) be the normalized value of individual \( i \) at age \( t \). Households also differ according to their preferences, denoted by \( J = H, L \). Households of type \( H \) have a high risk aversion (\( \rho_H \)) and high EIS (\( \psi_H \)), households of type \( L \) have low risk aversion (\( \rho_L \)) and low EIS (\( \psi_L \)). \( g_{i,t+1} = Y^p_{i,t+1}/Y^p_{i,t} \) is the
growth rate of the permanent component of income for the household.

\[ v_{i,t}^J(x_{it}, I_t = a) = (1 - \beta)c_{i,t}^{1-1/\psi_J} + \beta \left\{ E_t \left[ \xi_t(v_{i,t+1}(x_{i,t+1}, I_{t+1} = a))^{1-\rho_J}(Y_{i,t+1}^p/Y_{i,t}^p)^{1-\rho_J} + (1 - \xi_t)\varphi(x_{i,t+1})^{1-\rho_J}(Y_{i,t+1}^p/Y_{i,t}^p)^{1-\rho_J} \right] \right\}^{1-1/\psi_J} \]

\[ = \beta \left\{ E_t \left[ \xi_t(v_{i,t+1}(x_{i,t+1}, I_{t+1} = a) g_{i,t+1})^{1-\rho_J} + (1 - \xi_t)\varphi(x_{i,t+1}g_{i,t+1})^{1-\rho_J} \right] \right\}^{1-1/\psi_J}, \]

for \( a = 0, 1 \). The continuous state is \( x_{i,t} \) (normalized cash on hand) and its evolution is given by (8). The state also includes participation status (denoted by \( I_t \)) where 1 denotes participation and 0 denotes non-participation.

### Appendix C  Numerical Solution

We exploit the scale-independence of the maximization problem and rewrite all variables as ratios to the permanent component of labor income \((Y_{i,t}^p)\). The laws of motion and the value function can then be rewritten in terms of these normalized variables, and we use lower case letters to denote them. This normalization allows us to reduce the number of state variables to three: liquid wealth, participation status, and age. The problem is solved as follows.

For households who are already stock market participations, there is no
participation decision. Their value function is given by:

\[ v^J_t(x_{it}, I_t = 1) = \max_{c_t, \alpha_t^i, \alpha_t^b} \left\{ (1 - \beta)\bar{c}^{1-\psi_J} \right\} + \beta \left( E_t \left\{ \left( \frac{Y^p_{it+1}}{Y^p_{it}} \right)^{1-\rho_J} \left( \xi_t [v^J_{t+1}(x_{it+1}, I_{t+1} = 1)]^{1-\rho_J} + (1 - \xi_t) \varphi (x_{it+1})^{1-\rho_J} \right) \right\} \right) \]

Non-participants decide whether or not to incur the fixed cost at time (age) \( t \) (this is actually paid in the following period). They compare the two value functions associated with direct stock market participation or continued non-participation:

\[ v^J_t(x_{it}, I_t = 0) = \max_{0, 1} \{ v^J_t(x_{it}, I_t = 0), v^J_t(x_{it}, I_t = 1) \} \]

where \( I_t = 1 \) denotes stock market participation. The value of remaining a non-participant is given by:

\[ v^J_t(x_{it}, I_t = 0) = \max_{c_t, \alpha_t^i, \alpha_t^b} \left\{ (1 - \beta)\bar{c}^{1-\psi_J} \right\} + \beta \left( E_t \left\{ \left( \frac{Y^p_{it+1}}{Y^p_{it}} \right)^{1-\rho_J} \left( \xi_t [v^J_{t+1}(x^0_{it+1}, I_{t+1} = 0)]^{1-\rho_J} + (1 - \xi_t) \varphi (x^0_{it+1})^{1-\rho_J} \right) \right\} \right) \]

where

\[ x^0_{it+1} = \frac{x^b_{it+1}}{g_{i,t+1}} b_{i,t} + \frac{r^m_{t+1}}{g_{i,t+1}} m_{i,t} + U_{i,t+1} - \frac{\omega_{it}}{g_{i,t+1}} \]

is normalized cash-on-hand in period \( t + 1 \) conditional on the decision not to begin stock market participation at time \( t \).
For those who decide to participate

\[ v_t^J(x_{it}, I_t = 1) = \max_{c_t, \alpha_t^s, \alpha_t^b} \left\{ (1 - \beta) v_{t+1}^{1/\psi_J} + \beta \left( \frac{Y_{it+1}^p}{Y_{it}^p} \right)^{1-\rho_J} \left( \xi_t [v_{t+1}^J(x_{it+1}, I_{t+1} = 1)]^{1-\rho_J} + (1 - \xi_t) \varphi \left( x_{it+1}^J \right)^{1-\rho_J} \right) \right\} \]

where

\[ x_{it+1}^J = \frac{r_s}{g_{i,t+1}} s_{i,t} + \frac{r_b}{g_{i,t+1}} b_{i,t} + \frac{r_m}{g_{i,t+1}} m_{i,t} + U_{i,t+1} - \frac{\omega_{it}}{g_{i,t+1}} - \frac{F}{g_{i,t+1}} \]

is normalized cash-on-hand in period \( t + 1 \) conditional on the decision to begin stock market participation at time \( t \). The main difference between \( x_{it}^J \) and \( x_{it+1}^J \) lies in the fact that \( x_{it+1}^J \) includes returns from holding stocks \( \left( \frac{r_s}{g_{i,t+1}} s_{i,t} \right) \) but also includes the 'one-off' (normalized) cost of stock market participation \( \frac{F}{g_{i,t+1}} \).

We solve the model recursively backwards starting from the last period for households of each type \( J = H, L \). In the last period \( (t = T) \) the policy functions are trivial and the value function corresponds to the bequest function. We need to solve for four control variables in every year for stock-holders: current consumption \( (c_t) \), the fraction of the portfolio allocated to stocks \( (\alpha_t^s) \) and bonds \( (\alpha_t^b) \) (the fraction of saving allocated to money \( \alpha_t^m \) can be determined as the residual) and the participation decision. For every age \( t \) prior to \( T \), and for each point in the state space, we optimize using grid search. From the Bellman equation the optimal decisions are given as current utility plus the discounted expected continuation value \( (E_t v_{t+1}^J(\cdot)) \), which we can compute since we have just obtained \( v_{t+1}^J \). We perform all numerical integrations using
Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the risky asset returns. Cubic splines are used to perform the interpolation of the value function for points which do not lie on the state space grid, with more points used at lower levels of wealth where the value function has high curvature. Once we have computed the value of each alternative we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation, we obtain this period’s value function \( v_t^f(.) \), which is then used to solve the previous period’s maximization problem. This process is iterated until \( t = 1 \).
References


Figure 1: Baseline policy functions for Low EIS, Low RRA households (Solid line: stockholders, Dashed line: non-stockholders)
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(Solid line: stockholders, Dashed line: non-stockholders)
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